Dynamic Programming-based Heuristics for Markdown Pricing and Inventory Allocation of a Seasonal Product in a Retail Chain

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Abstract
A retailer wants to maximize her total expected profit from selling a seasonal product through her retail outlets. She does so by making two types of decisions: setting the markdown prices for retail outlets and deciding the inventory allocation to them. The number of price markdowns in a season is known before the season begins so does the distribution of the number of customers for a chosen price at a retail outlet. Inventories are stored at a distribution center. The retailer decides the amount of inventory allocated to each retail outlet right at each price markdown opportunity. There are inventory holding costs at the distribution center and the retail outlets; variable shipment cost for a unit transferred out from the distribution center; and the fixed ordering cost and the shortage cost at a retail outlet. The problem can be modeled as a dynamic program of multidimensional state space, which takes too heavy computational effort to solve. We develop dynamic programming-based heuristics to decide the price markdown and the inventory allocation. The heuristics take light computational effort and yet have good accuracy. Insights streamlining the operations of the retailer are deduced from the numerical results of the heuristics.

Keywords: Logistics; Markdown pricing; Inventory allocation; Heuristics; Supply chain management

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I. Introduction

Retailers mark down sale prices for promotion or clearance. The former is often a means to attract customers to boost up the customer demand; e.g., at Christmas. The clearance markdown is a sales gimmick for fashionable or perishable products with life times defined by trend (e.g., apparel in a retail outlet in a season), nature (e.g., fresh seafood in a wet market in a day), or technology (e.g., canned food in a supermarket in its shelf life). Such price markdowns are routine, recurrent, and generally planned ahead to maximize the expected total profit from a product. In particular, for apparel and fashion, there can be multiple price markdowns for a product within a season (Mantrala and Rao (2001), and Mohl (2004)).

This paper considers a single-product, multi-period markdown pricing problem together with inventory allocation decisions. The context of our problem can be regarded as an apparel retail chain which owns multiple (retail) outlets in a region. From historical data and experience, the regional manager knows the distributions of customer demands for the various prices possibly set at different times of the season for the outlets. Upon the arrival of goods at the distribution center (DC), the manager faces the problem of setting prices for goods and determining quantities to be shipped from the DC to the outlets. As shown subsequently, this problem can be modeled as a stochastic dynamic program, and the paper discusses its solution procedure.

As demonstrated by Lazear (1986), markdown pricing appears naturally when a retailer only knows the distribution of the reservation value (i.e., the valuation by customers) of a product, which is also the maximum price that a customer is willing to pay for the product. Lazear (1986) shows that the expected total revenue of a retailer from a product increases with the number of price markdowns to attract otherwise uninterested customers. In practice, markdown pricing has a long history; e.g., Pashigian (1988) traces the variation of markdown pricing of US department stores from 1925 to 1984. Among others, two striking
observations are that markdowns as a percentage of sales have been increasing after World War II, and that there is empirical evidence showing larger percentage markdowns for fashionable products. These results are further confirmed by Pashigian and Bowen (1991). Nowadays, a good back-of-the-envelop indicator for markdown may be 10% of the total cost (c.f. page 146 of Fung et al. (2008)). As shown in trade statistics, e.g., the total revenue of the global apparel industry in 2006 is of US$ 1,252.8 billion (fashionprodcuts.com (2010)), or the revenue of WalMart in 2009 is of US $401.2 billion (WalMart (2009)), there is a huge amount of money involved in price markdowns.

Early research works are on setting inventory levels or production quantities in order to find the best tradeoffs between revenue losses induced by price markdowns and by loss sales. Often there is a two-period model with the sales in the first period providing information such as the total demand in the two periods, the exact demand and the values of some system parameters in the second period, or the distributions instead of the exact values of some of these quantities (Bradford and Sugrue (1990), Fisher and Raman (1996), and Fisher et al. (2001)). As demand is price sensitive, naturally there are models that set optimal prices and ordering decisions by learning the demand parameter values or distributions from sales in earlier periods (Subrahmanyan and Shoemaker (1996), Petruzzi and Dada (2002), Aviv and Pazgal (2005), Lin (2006), Su (2007)). There are research works that determine the optimal dynamic prices based on the assumed forms of the prices and demand structures without demand learning (Smith and Achabal (1998), Feng and Gallego (1995), and Gupta et al. (2006)). In recent years, price markdowns are also used as means to coordinate supply chains (Tsay (2001), Lee (2007), Wang and Webster (2009)).

Our paper is related to Bitran et al. (1998) and Mantrala and Rao (2001). All three papers formulate stochastic dynamic programs for optimal pricing and inventory allocation of retail outlets under multiple price markdowns. Mantrala and Rao (2001) consider a single outlet problem, and Bitran et al. (1998) consider multiple outlets as we do. None of the two
papers in the literature has a DC that allocates inventory to outlets in every period as we do. In the first model of Bitran et al. (1998), inventory is allocated to outlets once at the beginning of the first period and further inventory transfer is forbidden. The second model of Bitran et al. (1998) allows inventory transfer among outlets at the beginning of each period. This is common for outlets in close vicinity; e.g., within the same city. However, for outlets belonging to the same operation district that are of long separation, it is more economical to keep and allocate inventory by a central DC without inventory transfer among outlets, which is the operations dynamics of our problem. Mantrala and Rao (2001) also disallow inventory transfer among outlets, and leftover inventory is disposed at the end of the planning horizon. In short, the first model of Bitran et al. (1998) is a special case of ours when inventory allocation is disallowed after the first period in our model; and the second model of Bitran et al. (1998) and ours consider different real-life operations dynamics.

There are differences in cost terms of the three papers. The first model of Bitran et al. (1998) considers only sales revenue, and the second model adds transfer cost to the first model. Our model considers both cost terms, with our shipment cost between the DC and outlets equivalent to the transfer cost in Bitran et al. (1998). In addition, our model considers fixed shipment cost and inventory holding costs in the DC and the outlets. Such cost terms are not in the models of Bitran et al. (1998), and these differences lead to new heuristics in this paper. The core set of cost terms in Mantrala and Rao (2001) is the same as ours. Some of their cost terms not explicitly considered by us can in fact be incorporated into ours by extending the scope of our cost terms.

Some papers on stochastic inventory spend effort on structural properties of the optimal policies. For problems in which the customer demand function varies wildly with price across periods, such structural properties may not exist, at least not the main focus; e.g., Mantrala and Rao (2001) solve their stochastic dynamic programs by numerical integration, and Bitran et al. (1998) solve by heuristics, both without considering any structural property.
In this paper, we solve small-size problems optimally, and develop heuristics H1, H2, and H3 for large-size problems. Heuristic H3 is from Bitran et al. (1998) for us to benchmark with literature. Our H1-heuristic is found to be accurate and with shorter computational time on average. Our numerical runs provide insights for the optimum operations of the system: (i) markdowns increase the system profit; (ii) the marginal contribution of inventory is of diminishing return with the nominal mean return, an index to set price; and (iii) the allocation of inventory in system highly affects the system profit.

For the rest of the paper, Section 2 describes first the problem and then the optimum dynamic programming (DP) formulation. Section 3 derives several heuristics. Section 4 reports the numerical experiments that show the effects of markdowns, inventory in system, and the allocation of inventory. Section 5 concludes the paper.

II. Problem statement and model description

In this paper, we study the optimal price markdowns and inventory replenishment of a seasonal product for outlets. How many markdowns should there be? As shown in Bitran and Mondschein (1997), for the optimal dynamic pricing policy and the policy with finite number of price markdowns, the difference in their expected profits is marginal, and decreasing, as the number of markdowns increases. Thus, our paper works with a finite, pre-determined number of markdowns, which is a practice widely adopted in real life.

There are different ways to model the effect of price on demand. Gallego and van Ryzin (1994) take the arrival rate of demands as an exponentially decreasing function of price. The common practice is to define the price-demand relationship through the reservation value with a known distribution; see, for examples, Bitran et al. (1998), Bitran and Mondschein (1997), Lazear (1986), and Lin (2006), where the first two papers further generate the demands of periods by Poisson demand rates and reservation values. Our model does not explicitly model the price-demand relationship this way. Instead, we assume that throughout
the season, at any price decision epoch, the manager picks prices from a *price menu*, which contains $L_{jk}$ types of prices $\bar{p}_{jk}^{(l)}$ for outlet $j$ at period $k$ together with its corresponding distribution $F_{jk}^{(l)}$ of customer demands, $1 \leq l \leq L_{jk}$, $1 \leq j \leq J$, $1 \leq k \leq K$. As expected, the demand decreases (the distribution $F_{jk}^{(l)}$ becomes stochastically smaller) as the price $\bar{p}_{jk}^{(l)}$ increases. See Mantrala and Rao (2001) for a similar price-demand relationship.

Our problem can be summarized as the following: A manager is responsible for the operations of a (central) $DC$ and $J$ outlets. The $DC$ keeps the inventory of a seasonal product, which is allocated to the outlets in the season. Naturally, whichever outlet sells more will be allocated a larger quantity of the product. While demand somewhat decreases with price, for any outlet at any period, the exact demand is a random function of price. For such an inventory allocation problem, the manager needs to consider the inventory holding costs in the $DC$ and outlets, the fixed and variable shipment costs from the $DC$ to outlets, and the shortage cost for loss sales in outlets for any short supply. There is no need to consider the re-allocation of inventory among outlets; the region is assumed to be of reasonable size so that re-allocating small quantities among the outlets is not worthwhile.

There are multiple opportunities to change price in the season, cutting the planning horizon into periods of *decreasing* (i.e., *non-increasing*) prices for outlets. Based on experience, the manager has already determined the number and the timing of (potential) price markdowns for the product. The prices of outlets are changed simultaneously, possibly to different values, at the beginning of a period. To set a price, the manager first checks the inventory in the $DC$ and the outlets. Then, simultaneously, he picks the price from the price menu for each outlet and decides the quantities of goods to be shipped from the $DC$ to the outlets. The decisions are based on (remaining) inventory levels of the $DC$ and the outlets, for the prior mentioned cost terms. Goods are delivered and received with zero lead time. The total inventory of an outlet is then used to satisfy the demands in the period. As
discussed before, goods are sent from the DC to outlets without any lateral transfer among outlets.

Suppose that there are $K$ periods in the season labeled from $K$ to 1 as time advances.

**System parameters:**

$h_j = \text{the unit inventory holding cost for venue } j, \text{ where for } j = 0 \text{ the venue is the DC, and for } 1 \leq j \leq J \text{ the venues are for the outlets; this convention of setting subscript } j \text{ to 0 for quantities related to the DC is adopted throughout the paper;}

$f_j = \text{the fixed shipment cost to ship a batch from the DC to outlet } j, 1 \leq j \leq J;\n
\quad c_j = \text{the unit shipment cost to ship an item from the DC to outlet } j, 1 \leq j \leq J;\n
\quad s_j = \text{the unit shortage cost of loss sales for outlet } j, 1 \leq j \leq J;\n
\quad D_{jk} = \text{the demand of the product at outlet } j \text{ at period } k, 1 \leq j \leq J, 1 \leq k \leq K.$

**Decision and intermediate variables:**

$I_{jk} = \text{the inventory at the beginning of period } k \text{ for venue } j, 0 \leq j \leq J, 1 \leq k \leq K;\n
\quad p_{jk} = \text{the price at period } k \text{ of outlet } j, 1 \leq j \leq J, 1 \leq k \leq K;\n
\quad y_{jk} = \text{the amount shipped from the DC to outlet } j \text{ at (the beginning of) period } k, 0 \leq j \leq J, 1 \leq k \leq K.$

Based on the symbols defined,

- the shipping costs of the distribution center at period $k = \sum_{j=1}^{J} f_j 1_{\{y_{jk}>0\}} + \sum_{j=1}^{J} c_j y_{jk}, 1 \leq k \leq K,$ where $1_{\{y_{jk}>0\}} = 1 \text{ if } y_{jk} > 0, \text{ and } = 0 \text{ otherwise.}$

- the inventory cost of the distribution center at period $k = h_0(I_{0k} - \sum_j y_{jk}), 1 \leq k \leq K;$

- the expected holding cost of outlet $j$ at period $k = h_j E\left(\frac{I_{jk} + y_{jk} + 0.5(I_{jk} + y_{jk} - D_{jk})}{2}\right),$ where the average inventory at the beginning and the end of a period is used to calculate the inventory cost, $1 \leq j \leq J, 1 \leq k \leq K;$

- the expected shortage cost of outlet $j$ at period $k = s_j E[D_{jk} - (I_{jk} + y_{jk})^+], 1 \leq j \leq J, 1 \leq k \leq K.$
\[ \leq K; \]

- the expected revenue of outlet \( j \) in period \( k \) is 
  \[ p_{jk} E[\min(D_{jk}, I_{jk} + y_{jk})], 1 \leq j \leq J, 1 \leq k \leq K. \]

At the beginning of any period \( k \), let \( I_k = (I_{jk}) \) be the vector of inventory on hand before ordering for the DC and the outlets; \( p_k = (p_{jk}) \) be the price vector eventually adopted by the outlets; \( y_k = (y_{jk}) \) be the quantity shipped to the outlets; and \( D_k = (D_{jk}) \) be the vector of demands of the outlets. By convention, \( y_{jk} \geq 0 \) for \( 1 \leq j \leq J \), and the amount shipped out from the DC, \( y_{0k} = -\sum_{j=1}^{J} y_{jk} \), is negative in value. At the beginning of period \( k \), the manager knows the prices \( p_{k+1} \) adopted in the last period and the current on hand inventory \( I_k \) of the DC and the outlets. He needs to decide \( p_k \) and \( y_k \) of period \( k \). Let \( V_k^*(p_{k+1}, I_k) \) be the maximum expected total profit from period \( k \) to period 1 of the whole region evaluated at the beginning of period \( k \), where the inventories on hand in the DC and outlets are represented by \( I_k \) and the prices \( p_{k+1} \) have been used at period \( k+1 \). For \( p_k \) and \( y_k \) adopted in period \( k \),

\[ I_{k-1} = [I_k + y_k - D_k]^+, K \geq k \geq 1; \]

\[ V_k^*(p_{k+1}, I_k) = \max_{p_{k+1} \geq p_k, I_{k-1} \geq \sum_{j=1}^{J} y_{jk}} \left\{ \sum_{j=1}^{J} p_{jk} E[\min(D_{jk}, I_{jk} + y_{jk})] - h_{0k}(I_{0k} - \sum_{j=1}^{J} y_{jk}) \right. \]

\[ - \sum_{j=1}^{J} f_j I_{(y_{jk} > 0)} - \sum_{j=1}^{J} c_j y_{jk} - \sum_{j=1}^{J} h_j E\left(\frac{(I_{jk} + y_{jk} - I_{0k} + \sum_{j=1}^{J} y_{jk})^2}{2}\right), \]

\[ - \sum_{j=1}^{J} s_j E[D_{jk} - (I_{jk} + y_{jk})]^+ \}

\[ + V_{k-1}^*(p_k, I_{k-1}) \]

\[ 1 \leq k \leq K. \]

The initial price \( p_{jk} \) can be any values in the price menu, and the initial inventory \( I_{jk} \) is defined by the on-hand inventory.
III. Heuristics

Haunted by the curse of dimensionality, high-dimensional dynamic programs are difficult to solve. Any significant reduction of computational time by structural properties of the optimal policy requires special dynamics in a problem, which is not assumed in relevant studies such as Mantrala and Rao (2001) and Bitran et al. (1998). Our study does not assume such special dynamics in the problem either.

Over the years, many approximation approaches have been developed to solve dynamic programs, including the certainty equivalent controllers, the open-loop feedback control schemes, the look-ahead approaches, the approximations of $V_{k-1}(\cdot)$ by convexity, linearity, and special functional forms of $V_{k-1}(\cdot)$, and the discretization approaches of state spaces and decision sets. Interested readers can refer to Bertsekas (2007) for a summary of approximate schemes of dynamic programs. Here we mention Bertsekas and Castanon (1999) and Herbots et al. (2010) as examples of a rollout algorithm.

Instead of solving the price setting and inventory allocation decisions simultaneously, Bitran et al. (1998) determine the two decisions iteratively: one of the two sets of decisions is fixed in turn while the optimal of the other set of decisions is sought; the iteration continues until the two sets of decisions converge. Such an idea is common for solving complicated problems of heavy computational effort; see, e.g., Hwe et al. (2006) fix in turn the frequencies of merged bus routes and the routes to be merged in regrouping bus routes. Our heuristics follow the same spirit. For any period $k$, the price vector $p_k$ and the inventory vector (i.e., the vector of inventory reservation $A_k$ to be defined in Section III.2) are fixed in turn while the optimal value of the other vector is sought. This iterative process continues till both the price vector and the inventory vector converge. While the solution structures of Bitran et al. (1998) and this paper are common in this regard, the detailed optimization problems for each iteration of the two papers are very different from each other. Moreover,
our H1-heuristic estimates the potential contribution of an item in the next period when the item is allocated in the current period, a feature absent from heuristics in Bitran et al. (1998).

To benchmark the performance of H1-heuristic, we also develop H2-heuristic and H3-heuristic. In terms of idea, the two heuristics are successively closer to the heuristics in Bitran et al. (1998) than H1. H2 applies H1 to a two-period problem formed by lumping all future periods into a super-period. Other than the different cost terms considered in the detailed model, H3 is basically a heuristic from Bitran et al. (1998).

The organization of the subsections in this section is as follows. Subsection III.1 introduces the IPV heuristic, which estimates the optimal inventory vector from the given price vector. Our IPV heuristic also considers the value of an item if it is left for the next period, an issue not taken up by the relevant studies. Subsection III.2 discusses the PMD heuristic, which estimates the optimal price vector from the inventory vector. Subsection III.3 combines the IPV and PMD heuristics into H1-heuristic, which is the main result of the paper. Finally, Subsection III.4 discusses other two heuristics H2 and H3, which are used in benchmarking with the performance of H1.

III.1. Inventory allocation from price vectors: the IPV heuristic

The IPV heuristic determines the inventory allocation for a price vector. First define

\[
g_{jk}(y) = p_{jk}E\left[\min(D_{jk}, y)\right] - h_jE\left[\frac{y + \left[y - D_{jk}\right]_+}{2}\right] - s_jE\left[D_{jk} - y\right]_+ , \quad p_{jk} \in P_j, \tag{3}
\]

for each feasible price \(p_{jk}\). (\(D_{jk}\) is a function of \(p_{jk}\).) The term \(p_{jk}E[\min(D_{jk}, y)]\) is the expected revenue of outlet \(j\) with \(y\) units on hand and price \(p_{jk}\); the term \(h_jE\left[\frac{y + \left[y - D_{jk}\right]_+}{2}\right]\) is the expected inventory cost by taking the average inventory at the beginning and the end of the period; and the term \(s_jE\left[D_{jk} - y\right]_+\) is the expected shortage cost. Thus, \(g_{jk}(y)\) is the expected profit of outlet \(j\) if
she adopts price $p_{jk}$ with $y$ unit on hand for the period, *without* considering the fixed and variable ordering cost terms. Similarly, define

$$G_{jk}(y) = g_{jk}(y) - cy, \quad p_{jk} \in \mathbb{P},$$

that also considers the variable shipment cost.

**Lemma 1.** The function $g_{jk}(y)$ and $G_{jk}(y)$ are concave in $y$ for all $j$.

**Proof.** For any constant $d$, $\min(d, y)$ is a concave function of $y$; $(y-d)^+$ and $(d-y)^+$ are convex functions of $y$. The expectation operation preserves concavity and convexity. Thus, $E[\min(D_{jk}, y)]$ is a concave function of $y$, and $E(y-D_{jk})^+$ and $E(D_{jk}-y)$ are convex functions of $y$. The assertions then follow because positive scaling and summation preserve concavity and convexity, and negation flips a convex function concave.

Let $M$ be a positive integer. Suppose that we need to assign $M$ items to the outlets in period $k$ to maximize the expected total profit when the ordering costs are ignored. Effectively we solve the optimization problem $OPT$:

$$\max \sum_j (g_{jk}(y_{jk}) - g_{jk}(0)),
\text{s.t. } \sum_j y_{jk} = M;
\quad y_{jk} \in \{0, 1, 2, \ldots\}.$$  

Consider the scheme to assign the $M$ items one by one to the outlets. Suppose that at a generic step, outlet $j$ has already held $y_j$ items. The *marginal increase* in objective function by adding one item to outlet $j$ is $g_{jk}(y_j+1) - g_{jk}(y_j)$.

**Lemma 2.** $OPT$ is maximized by the myopic itemized allocation scheme to assign each item to the outlet of the highest marginal increase in the objective function. Any tie can be broken arbitrarily.

**Proof.** Rewrite $g_{jk}(y_{jk}) - g_{jk}(0) = [g_{jk}(y_{jk}) - g_{jk}(y_{jk}-1)] + [g_{jk}(y_{jk}-1) - g_{jk}(y_{jk}-2)] + \ldots + [g_{jk}(1) - g_{jk}(0)]$. The concavity of $g_{jk}$ ensures that $g_{jk}(y_j+1) - g_{jk}(y_j)$ is decreasing in $y_j$, for all $j$, for all
Thus, adding items in the decreasing values of \([g_{jk}(y_j+1) - g_{jk}(y_j)]\) maximizes the objective function, which is the myopic itemized allocation scheme.

From here onwards we refer to the myopic scheme as the rule of maximal expected marginal return (MEMR). The optimality of of MEMR in Lemma 2 relies on the concavity of \(g_{jk}(\cdot)\). The concavity of \(G_{jk}(\cdot)\) from Lemma 1 ensures that the assertion for \(g_{jk}(\cdot)\) in Lemma 2 also holds for \(G_{jk}(\cdot)\).

However, for two reasons, allocating inventory purely by MEMR does not lead to an optimal solution for our problem. First, Eq. (3) ignores the fixed and variable shipment costs. Second, the result in Lemma 2 is for one period whereas our problem is of multiple periods. To account for these two effects, we develop a heuristic that still considers the expected marginal return in allocating items one by one. However, the allocation is further governed by the following three principles (C1)–(C3).

(C1). In considering the contribution of an item towards the objective function, both its contributions in the current period and in the future are considered. The length of the future to consider is a parameter of the heuristic.

(C2). For an outlet, if (further) adding an item to it does not increase the expected profit, the outlet is illegible to receive any item. The shipment costs are considered in the eligibility of outlets.

(C3). Items are assigned one by one to the eligible outlets according to MEMR.

Define

\[
\beta_{j,k}(l) = g_{j,k}(l) - g_{j,k}(0), \quad 1 \leq j \leq J, 1 \leq k \leq K, l \geq 1.
\]  

\(\beta_{j,k}(l)\) is the expected profit for outlet \(j\) to start period \(k\) with \(l\) items rather than with zero inventory, where \(l \geq 1\). In the same spirit, define

\[
\delta_{j,k}(l) = g_{j,k}(l) - g_{j,k}(l-1), \quad 1 \leq j \leq J, \quad 1 \leq k \leq K, \quad l \geq 1.
\]
\( \delta_{j,k}(l) \) is the marginal expected return of the \( l \)th items for outlet \( j \) in period \( k \), no matter whether the item comes from the outlet’s initial inventory or the DC’s. Thus, \( \delta_{j,k}(l) \) is also a measure of the value of leaving an item from period \( k+1 \) to period \( k \) for outlet \( j \) when the outlet holds \( l-1 \) items in period \( k \); \( \delta_{j,k}(0) \equiv 0 \). Let \( |D_{jk}| \) be the largest possible integer value taken by \( D_{jk} \), where \( D_{jk} \sim \{q_{jk,s}\} \), i.e., \( P(D_{jk} = s) = q_{jk,s}, s = 0, 1, \ldots, |D_{jk}| \). Let \( \gamma_{jk}(x) \) be the expected revenue of outlet \( j \) in period \( k \) of having \( x \) units of inventory on hand.

\[
\gamma_{jk}(x) = g_{jk}(x) + \sum_{s=0}^{x-1} q_{jk,s} \beta_{j,k-1}(x-s), 1 \leq j \leq J, 1 \leq k \leq K, l \geq 1.
\]

(7)

Therefore, for an outlet with \( I_{jk} \) units of inventory on hand in period \( k \), the expected profit of ordering \( y \) units is computed by

\[
\gamma_{jk}(I_{jk} + y) - f - yc + yh_b.
\]

(8)

Items are allocated one by one to the periods of the outlets. For ease of reference, we call each of these outlet and period pairs an outlet-period in the subsequent discussion. At any stage of allocation, an outlet is illegible for further allocation to a period if for any step, adding an item to the period for the outlet reduces the expected profit of the period as calculated from Eq. (8). Among the eligible outlets, an item is assigned to the outlet that maximizes

\[
\theta_{jk}(I_{jk} + y + 1) = g_{jk}(I_{jk} + y + 1) - g_{jk}(I_{jk} + y) + \sum_{s=0}^{\min(I_{jk} + y, |D_{jk}|)} q_{jk,s} \delta_{j,k-1}(I_{jk} + y - m), 1 \leq j \leq J, 1 \leq k \leq K, l \geq 1.
\]

(9)

In the following algorithm, let \( A_{jk} \) be the inventory reservation of outlet \( j \) at period \( k \), i.e., it is the amount of inventory in \( I_{jk} \) and \( I_{0k} \) reserved for outlet \( j \) at period \( k \). The values of \( A_{jk} \) are from the results of the PMD heuristic covered in the next section.

**Algorithm 1.** The IPV heuristic

1° Note the result of \( A_{jk} \) from the PMD heuristic, \( 1 \leq j \leq J, 1 \leq k \leq K-l \).

2° Compute \( \gamma_{jk}(A_{jk}+1) \) from Eq. (7), \( 1 \leq j \leq J, 1 \leq k \leq K \), to determine the set of eligible outlet-periods.
Stop if there is no eligible outlet-period, or there is no inventory in the DC; else:

3.1° For all eligible outlet-period, compute \( \theta_{jk}(A_{jk}+1) \) from Eq. (9) with \( A_{jk} \) taking the role of \( I_{jk} + y \).

3.2° Assign one DC item to the outlet-period that maximizes \( \theta_{jk} \); break ties arbitrarily.

3.3° Set \( A_{jk} = A_{jk} + 1 \). Update \( \gamma_{jk}(A_{jk}+1) \) for the outlet-period with the most recent item allocation. Check its eligibility and go back to 3°.

III.2. Prices from mean demands: the PMD heuristic

The PMD heuristic determines the optimal price vector \( p_k \) from inventory reserved for output-periods. Let \( A_k = (A_{1k}, \ldots, A_{Jk}) \), be the vector of inventory reservation at period \( k \), where \( A_{jk} \) is the amount of inventory in \( I_{jk} \) and \( I_{0k} \) reserved for outlet \( j \) at period \( k \). The inventory reservation is defined for a given price vector \( p_k \). Let \( \bar{d}_{jk} = E(D_{jk}) \). (Again, \( A_k \) and \( \bar{d}_{jk} \) are functions of \( p_{jk} \) though we omit \( p_{jk} \) for simplicity of exposition.) The distribution of \( D_{jk} \) can be derived from the reservation value on a Poisson arrival stream of customers as in Bitran et al. (1998). However, our subsequent heuristics do not depend on the Poisson assumption. In the following, we calculate \( A_k \) for \( k = K \) to 1:

\[
\text{If } \sum_{k=K}^1 \bar{d}_{jk} \leq I_{jk} \text{ for all } j, \text{ set } A_{jk} = I_{jk};
\]

\[
\text{else set } A_{jk} = I_{jk} + \frac{\left[ \sum_{k=K}^1 \bar{d}_{jk} - I_{jk} \right]^+}{\sum_{j=1}^J \sum_{k=K}^1 \bar{d}_{jk} - I_{jk}} I_{0k},
\]

Let \( \tau \) be the sum of the fractional parts of \( A_{jk} \); \( \tau \) is an integer, if not zero. Then round to integers for all \( A_{jk} \), up for the first \( \tau \) largest fractional parts of \( A_{jk} \), and down for the rest.

Roughly speaking, Eq. (10) reserves the initial inventory of the distribution center in proportional to the excess of the total mean demands of an outlet over its initial inventory. The PMD heuristic then searches for the optimal price vector \( (p_{jk}, \ldots, p_{J1}) \) for an outlet based
on the following two conditions:

(C4). The mean demands are used in place of the stochastic demands to estimate various monetary terms.

(C5). An outlet first uses the initial inventory \( I_{jk} \) to satisfy the demand. At the period that \( I_{jk} \) has been used up, the amount reserved for the outlet from \( I_{0k} \) is sent to the outlet.

The inventory reservation \( A_k \) records inventory available for the outlets. In general,

\[
A_{j,k-1} = \left[ A_{jk} - \bar{d}_{jk} \right]^+, \quad k = K, \ldots, 2. \tag{11}
\]

To estimate the amount of inventory on hand, note that at the beginning of period \( K \), \( I_{jk} \) items are in outlet \( j \) and another \( A_{jK} - I_{jk} \) are reserved in the DC for the outlet. Let \( \kappa_j \) be the first period that outlet \( j \) is out of inventory, that is,

\[
\sum_{k=K}^{\kappa_j+1} \bar{d}_{jk} \leq I_{jk} < \sum_{k=K}^{\kappa_j} \bar{d}_{jk}. \tag{12}
\]

Then for periods \( K \) to \( \kappa_j +1 \),

\[
I_{jk} = I_{j,k+1} - \bar{d}_{j,k+1}, \quad k = K - 1 \text{ to } \kappa_j + 1; \tag{13}
\]

and for periods \( \kappa_j \) to 1, the inventory levels at the beginning of a period for outlet \( j \) are

\[
I_{jk} = \left[ A_{jk} - \sum_{k=K}^{\kappa_j+1} \bar{d}_{jk} \right]^+, \quad \text{and} \tag{14}
\]

\[
I_{jk} = \left[ I_{j,k+1} - \bar{d}_{j,k+1} \right]^+, \quad k = \kappa_j - 1 \text{ to } 1.
\]

With the above estimates for the inventory on hand, we can estimate the costs across the periods for the outlets and DC for any given collection of price vectors for outlets. The price vector across the periods that maximizes the profit is taken as the optimal prices for the outlets. The collection of the prices for the current period from the optimal price vectors is taken as the optimal price vector of the current period.

For any feasible price vector \( (p_{jK}, \ldots, p_{j1}) \), the estimated revenue in period \( k \) for outlet \( j \) is
\[ p_{jk} \min \left( A_{jk}, \overline{A}_{jk} \right). \] The estimated total revenue for outlet \( j \) in the planning horizon is

\[ v_j = \sum_{k=K}^1 p_{jk} \min \left( A_{jk}, \overline{A}_{jk} \right); \tag{15} \]

the estimated total shortage cost for outlet \( j \) is

\[ \sum_{k=K}^1 \left[ \overline{d}_{jk} - A_{jk} \right]^+; \tag{16} \]

the estimated total shipping cost for outlet \( j \) is

\[ \sum_{k=K}^1 f_j \left[ \overline{d}_{jk} - I_{jk} \right]^+ + \sum_{k=K}^1 c_j \left[ \overline{d}_{jk} - I_{jk} \right]^+; \tag{17} \]

and the estimated inventory cost of outlet \( j \) is

\[ \frac{1}{2} h_j \sum_{k=K}^1 \left( I_{jk} + I_{j,k-1} \right). \tag{18} \]

Note that \( A_{jk} - I_{jk} \) units are stored in the DC for periods \( K \) to \( \kappa_j + 1 \). Thus, the inventory cost of outlet \( j \) in the DC is

\[ h_j \left( A_{jk} - I_{jk} \right) \left( K - \kappa_j \right). \tag{19} \]

The estimated profit of outlet \( j \) for a given pair of reservation \( A \) and price vector is

\[ (15)-(16)-(17)-(18)-(19). \tag{20} \]

Eq. (14) to (20) estimate the profit for outlet \( j \) for one price vector. The feasible prices are decreasing across periods, and there are only a finite number of them. In practice, it is simple to round up all feasible prices vectors for an outlet. For example, suppose that there are three periods in the planning horizon and there are two prices \( p_2 > p_1 \). Then the set of feasible prices is \{\( p_2, p_2, p_2 \), \( p_2, p_2, p_1 \), \( p_2, p_1, p_1 \), \( p_1, p_1, p_1 \}\}. We skip the detail to round up the feasible price vectors.

**Algorithm 2.** The PMD heuristic

1° Round up the feasible price vectors for an outlet in a given period.

2° For each outlet calculate the expected profit for each feasible price vector from Eq. (20).

The optimal price vector is the one that maximizes the expected profit for the outlet.
III.3. Heuristic H1

This section combines the IPV and PMD heuristics developed in earlier sections into H1-heuristic that determines both the prices and the inventory allocation of outlets for the current period. In application, H1 is executed in the rolling horizon fashion to make decisions for outlets for the current period after finishing with the inventory transactions of the last period.

Algorithm 3. The H1-heuristic

1° Set \( p_j = (p_{jK}, \ldots, p_{jK}) \) for all \( j \). Set \( A \) according to Eq. (10), possibly with rounding.

2° Given \( A \), find the new optimal price vectors by the PMD heuristic from Algorithm 2. Go to 4° if for all \( j \) the new optimal price vector remains the same as \( p_j \); else set \( p_j \) to be the new optimal price vector and proceed to 3°.

3° Given \( p_j \), use the IPV heuristic from Algorithm 1 to determine the optimal inventory allocation to outlets in the periods. Proceed to 4° if the new inventory allocation \( A \) is the same as the current optimal inventory allocation; else proceed to 2° with this \( A \).

4° Set the price of outlet \( j \) to \( p_{jK} \), the first element of \( p_j \). Order if \( \max(d_{jk}(p_{jK}), A_{jk}) > I_{jk} \).

In case of ordering, for \( h_j \leq h_0 \), order \( A_{jk} - I_{jk} \); for \( h_j > h_0 \), order

\[
\max\left(d_{jk}(p_{jK}) - I_{jk}, A_{jk} - I_{jk}, f_j \frac{(h_j - h_0)}{}\right).
\]

In step 4°, an outlet orders only if both the mean demand and the inventory allocation are higher than her inventory on hand. In case of placing an order, if the inventory holding cost at the DC is no less than that of outlet \( j \), send in as much inventory as IPV indicates to be optimal; else the storage costs and ordering cost need to be considered.

III.4. Other heuristics

In its calculation, H1 estimates the possible contribution of an item from the very next period. It is possible to estimate the effect for all future periods. To do so, the demands of
all the future periods \( K - 1 \) to 1 are lumped together into that to form a “super-period”. The price set in the super-period practically says that the prices of all periods after \( K - 1 \) are in fact of the same price. The mean demand of the super-period is the sum of the mean demands of all the future periods. The whole procedure of applying H1-heuristic to the two-period problem with the super period is H2-heuristic.

Other than H1 and H2 developed in this paper, we further consider H3-heuristic which does not consider any contribution from future periods. This heuristic is close in spirit to those in Bitran et al. (1998), though our problem considers cost terms ignored by Bitran et al. (1998). Furthermore, in H3, we try different inventory starting conditions, leading to variants H3_best, H3_max, and H3_min. An arbitrary set of initial prices can be used in Eq. (10) to calculate \( A_{jk} \) for H3. Among all the feasible prices, we may choose the maximum in each period (H3_max) or the minimum (H3_min) in each period. Our results indicate neither of these two variants can dominate the other one. Therefore, we design a hybrid version that picks the best among all the feasible prices (H3_best). In this hybrid method, each eligible price for the current period is used to generate the reservation amount calculated by Eq. (10). The corresponding profit based on each of the prices is calculated before the selection can be made. Intuitively, this procedure can lead to a better decision in each period. However, it does spend more run time due to the comparison effort.

IV. Computational results

Bitran et al. (1998) calibrate their heuristics in two sets of numerical experiments. The first set of experiments is of two-outlet, five-period to compare the accuracy of their heuristics HEUR1, HEUR2, and HEUR3 for various initial inventory allocations. The second set of experiments is of three-outlet, three-period to check the accuracy of their heuristic HEUR4 for various total amounts of initial inventory in system. We have carried out many numerical experiments for various multi-period single-, double-, and triple-outlet systems similar to
Bitran et al. (1998). The following results calibrate the performance of our heuristics and to explain the properties of the inventory systems.

In all subsequent numerical experiments, an outlet spends $6 to place an order, and $6 to transport an item from the DC to the outlet. An outlet pays $1 for each item carried over to the next period. For multi-period experiment instances, this inventory holding cost is prorated according to the number of periods so as to compare experiment instances of different number of periods under the same footing; e.g., for a 4-period instance, the inventory holding cost is $0.25 per item per period. The unit holding cost of the DC is of $0.2 per item for a single-period instance, lower than that for an outlet, and is again prorated for multi-period instances. The shortage cost of each unit is $1. Throughout our experiments, the price menu is {12, 18, 24, 30}. For each price, it is possible to generate the demands according to the reservation values for Poisson distributions as in Bitran et al. (1998). We adopt reservation values that follow the uniform distribution, i.e., for mean demand rate to be \( \lambda \) at the lowest price $12, the mean demand rates for prices $18, $24, and $30 are 0.75\( \lambda \), 0.5\( \lambda \), and 0.25\( \lambda \), respectively. As the value of \( \lambda \) can only be attained for the lowest price in the price menu, we refer it to the ceiling arrival rate.

Consider any given distribution of demand \( D \). Approximate the distribution by a discrete distribution consisted of \( m \) point masses \( w_1, w_2, \ldots, w_m \) with probabilities of \( q_1, q_2, \ldots, q_m \), respectively. The point masses and the probabilities are defined by break points \( b_0 (= 0), b_1, \ldots, b_g (= \infty) \) that partition the support of \( D \) such that

\[
q_s = \sum_{b_{s-1} \leq b_i < b_s} P(D = i), \quad s = 1, \ldots, m,
\]

where \( b_0, \ldots, b_g \) are chosen to make \( q_s \) approximately of the same value; and

\[
w_s = \frac{\sum_{b_{s-1} \leq b_i < b_s} iP(D = i)}{q_s}, \quad s = 1, \ldots, m.
\]

The construction works for any value of \( m \), and can in fact approximate any distributions. In
our numerical experiments we choose $m = 2$ to approximate Poisson distributions.

The expected cost of the exact solution is obtained from solving the dynamic program specified in Eq. (1) and (2). The performance of a heuristic is obtained by taking its average performance over 1,000 replications for each parameter setting. In each replication, heuristics are applied in the rolling horizon fashion till the end of the planning horizon. The monetary terms of all periods are summed up to find the net profit of the instance.

All the computational experiments are conducted in a computer with Intel Core 2 Quad CPU Q6600, 2.4 GHz and 1.98 GB of RAM.

IV.1. Accuracy of the heuristics and the effect of the number of markdowns

We first calibrate the heuristics’ performance by a one-outlet system for one to five periods with the DC inventory held fixed. The effect of the number of markdowns is obtained as a by-product. Set the arrival rate of the $k$th period $\lambda_k$ to $\eta^{K-k}\lambda/\sum_{j=0}^{K-1}\eta^j$, $\eta \in (0, 1)$, $k \in \{1, \ldots, K\}$ for a $K$-period problem. By this construction, the total ceiling arrival rate of all periods $= \lambda = \sum_{k=1}^{K} \lambda_k$, and for the same price, the arrival rate decreases across the periods. In numerical runs, we set $\lambda = 320$ and $\eta = 0.8$, i.e., $\lambda_k = (0.2)^{0.8^{K-1}}(320) / 1 - 0.8^k$. For other parameters, $I_{0K} = 160$ and $h_0 = $0.2/$K$; $I_{jK} = 0$ and $h_j = $1/$K$ for $1 \leq j \leq J$. See Table 1 for expected profits, the gaps in expected profits, and the run times of the heuristics and DP. Note that the instances for the $i$-period, $j$-outlet experiment are labeled as $PiRj$.

Table 1 confirms the intuition that the expected profits is increasing with the number of markdowns. As expected, the marginal increase in expected profit decreases as the number of markdowns increases. As shown in Table 1, H1 performs best among all the heuristics on average. It is accurate (on average 0.34% less than the maximum profit) and quick (on average less than one-fifth of the computation time of DP). In general, the more the number of outlets and the more the number of periods, the more saving in computational time H1 has.
Table 1
Comparison of profit, profit gap, and run time for one-outlet instances

<table>
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<th>Exp. 1</th>
<th>P: Profit, G: Gap (%), and R: Run time (minute)</th>
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<td>R</td>
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<td>P2R1</td>
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</tr>
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<tr>
<td>R</td>
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</tr>
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<td>P3R1</td>
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<tr>
<td>P4R1</td>
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<td>R</td>
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</tr>
<tr>
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<tr>
<td>R</td>
<td>4.76</td>
</tr>
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IV.2. Accuracy of the heuristics and the effect of the initial DC inventory

In Experiment 2, we extract our results of P3R2 to demonstrate the accuracy of the heuristics and the effect of the initial DC inventory. For the two outlets, we maintain the same ratio \( \lambda _j^k = (0.8)^{3-k} \lambda _j^1 \), for outlet \( j \in \{1, 2\} \), for period \( k \in \{1, 2, 3\} \), and that the total ceiling arrival rates of outlets 1 and 2 in the three periods are 40 (obtained from \( \sum_{k=1}^{3} \lambda _1^k \)) and 30 (obtained from \( \sum_{k=1}^{3} \lambda _2^k \)), respectively. Our initial DC inventory changes from 20 to 50 (in step of 5) in this set of experiments. The expected profit of DP and the heuristics, the gaps between the expected profits of the heuristics and the DP, and the run times of all the approaches are shown in Table 2 for this set of experiments.

By construction, the total mean demand rates at prices $12, $18, $24, and $30 are \( \lambda \) (\( \lambda = 70 \)), 0.75\( \lambda \), 0.5\( \lambda \), and 0.25\( \lambda \), respectively. Define the nominal mean return as the product.
of the mean demand rate and its price. Then the nominal mean returns of the four prices $12, $18, $24, and $30 are $12\lambda$, $13.5\lambda$, $12\lambda$, and $7.5\lambda$, respectively. In some sense we would expect that the second price, $18, which corresponds to the mean demand rates 52.5, is a reasonably good choice of price. Given this contemplation, we would expect that increasing the initial DC inventory from 20 to 50 will increase the expected profit, which is confirmed by the column for DP in Table 2. Note that the marginal increase in expected profit decreases with the increase in inventory. Beyond a point the DC inventory cannot be consumed by the demands of the selected price and the expected profit will decrease.

As shown in Table 2, the performance of H1 is the best. The gap is within 2% of the true optima for 35 or more units of initial DC inventory. Its overall performance is within 3.55% of the true optima taking the average across all initial DC inventory.

The number of states of the DP is an exponential function of the initial inventory in system, and the run time increases accordingly. For successively smaller percentage of increase in the initial inventory, the run time of DP increases rapidly (see Table 2). On the other hand, the run times of the heuristics are short. Heuristic H1, with the best performance among the heuristics, consistently takes a very small fraction of run times of DP.
Table 2
Comparison of profit, profit gap, and run time of P3R2 with different DC inventory

<table>
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<tr>
<th>Exp. 2 Initial inv.</th>
<th>P: Profit, G: Gap (%), and R: Run time (minute)</th>
<th>DP</th>
<th>H1</th>
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<th>H3_best</th>
<th>H3_max</th>
<th>H3_min</th>
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<td></td>
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<td>3.25</td>
<td>4.13</td>
<td>0.54</td>
<td>0.15</td>
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IV.3. Accuracy of the heuristics and the effect of the distribution of inventory

Experiment 3 deals with a two-period, three-outlet instance. Our preliminary experiment indicates that the computer memory limits the system inventory to be 23 units or less for DP to run successfully. Thus, we choose a total of 20 units, allocating among the DC and the three outlets in 7 scenarios (Table 3). As for the three outlets, the ceiling arrival rates of outlets 1, 2, and 3 in the two periods are 20, 15, and 10, respectively. These ratios of 4:3:2 hold for the total mean demands as well as the mean demands of individual periods. The expected profit of DP and the heuristics, the gaps between the expected profits of the
heuristics and the DP, and the run times of all the approaches are shown in Table 4 for this experiment.

Table 3

<table>
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<tr>
<th>Scenario</th>
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<th>R2 inventory</th>
<th>R3 inventory</th>
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</tbody>
</table>

Table 4 reflects the importance of inventory allocation for the profit of a company. In this experiment, the ceiling of total arrival rate equals 45 (obtained from 20+15+10). For the four prices $12, $18, $24, and $30, prices $18 has the highest nominal mean return. However, the price corresponds to a mean demand of 33.75 (obtained from 0.75*45), which is higher than the total inventory available. Thus, the nominal mean returns suggest a price around $24, whose total mean arrival rate is 22.5, a little bit over the total inventory available. Given this setting, we would expect that the optimal allocation of inventory should be close to, but not necessarily exactly, the mean demands of the outlets. Such an allocation of inventory corresponds to $\frac{80}{9}$, $\frac{20}{3}$, and $\frac{40}{9}$ units among the three outlets. The optimal inventory allocation is determined by the distributions, not only the means, of the demands. However, the means of demands capture the first-order effect of the allocation. Table 4 confirms that any allocation of inventory that violates this general guideline would not be desirable. Consider the first three scenarios all with 5 units in the DC and different allocations of inventory in the outlets. None of them is close to the 4:3:2 ratios. However, the inventory allocation (5/5/5/5) has better chance to match the ratios ($\frac{80}{9} : \frac{20}{3} : \frac{40}{9}$) than the allocation (5/10/5/0), which in turn has better chance than the allocation (5/15/0/0). As reflected from
the results of DP, indeed the allocation (5/5/5/5) has the largest expected profit and (5/15/0/0) the smallest profit among the three scenarios. The same phenomenon occurs at the two scenarios with 10 units in the DC. The inventory allocation (10/5/5/0) indeed has larger expected profit than the allocation (10/10/0/0).

Table 4

Comparison of profit, profit gap, and run time of P2R3 with different initial inventory allocation

<table>
<thead>
<tr>
<th>Exp. 3</th>
<th>Inv. of DC/R1/R2/R3</th>
<th>P: Profit, G: Gap (%), and R: Run time (minute)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DP</td>
<td>H1</td>
</tr>
<tr>
<td>5/5/5</td>
<td>P 253.67</td>
<td>253.79</td>
</tr>
<tr>
<td></td>
<td>G 0.00</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>R 115.33</td>
<td>8.29</td>
</tr>
<tr>
<td>5/10/5</td>
<td>P 228.20</td>
<td>227.88</td>
</tr>
<tr>
<td></td>
<td>G 0.00</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>R 115.03</td>
<td>9.51</td>
</tr>
<tr>
<td>5/15/0</td>
<td>P 161.29</td>
<td>159.84</td>
</tr>
<tr>
<td></td>
<td>G 0.00</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>R 115.18</td>
<td>8.34</td>
</tr>
<tr>
<td>10/5/5</td>
<td>P 220.50</td>
<td>220.34</td>
</tr>
<tr>
<td></td>
<td>G 0.00</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>R 437.04</td>
<td>11.14</td>
</tr>
<tr>
<td>10/10/0</td>
<td>P 194.54</td>
<td>195.49</td>
</tr>
<tr>
<td></td>
<td>G 0.00</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>R 436.05</td>
<td>12.81</td>
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<tr>
<td>15/5/0</td>
<td>P 186.84</td>
<td>187.70</td>
</tr>
<tr>
<td></td>
<td>G 0.00</td>
<td>0.46</td>
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<tr>
<td></td>
<td>R 723.00</td>
<td>14.78</td>
</tr>
<tr>
<td>20/0/0</td>
<td>P 155.05</td>
<td>150.84</td>
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<tr>
<td></td>
<td>G 0.00</td>
<td>2.72</td>
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<td></td>
<td>R 793.60</td>
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</tr>
<tr>
<td>Average</td>
<td>P 200.01</td>
<td>199.41</td>
</tr>
<tr>
<td></td>
<td>G 0.00</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>R 390.75</td>
<td>11.94</td>
</tr>
</tbody>
</table>

As observed from Table 4, H1 and H2 show very good performance, both being within 1% gap from the optima, with H2 this time slightly better than H1. Table 4 indicates the number of states increases exponentially with the number of outlets and the amount of
inventory. Thus, DP takes more than 12 hours to compute in some scenarios where H1 take less than 3% of the run time of DP. Variants of heuristics H3 are quick, though their accuracy is not as good as H1.

V. Conclusions

In this paper we have constructed heuristics to set the markdown prices and inventory replenishment decisions for multiple outlets. For a problem such that the number of states increases exponentially with the amount of inventory in system and the number of outlets, our heuristics either take insignificant computational time by themselves, or insignificant fractional computational time of the optimal DP. The heuristic H1 developed in our study has shown to be accurate in general and with short computational time.

Other than identifying the best heuristics among the constructed ones, our derivation of the heuristics and the numerical experiments provide four insights for the operations of outlets (and in general of retailers). First, markdown is a tool for outlets to increase their profit. However, the marginal increase in profit decreases as the number of markdowns increases, and theoretically there is a finite upper bound value even if there are an infinite number of markdowns. Note that our objective function excludes the effort spent in and the reaction of consumers to markdowns. Thus, outlets must be careful in choosing the number of markdowns in a sale season. Second, the nominal mean return is a handy index to decide the price to set. It captures the first-order effect to set price, which can serve as a rule-of-thumb for outlets that are not familiar with analytical manipulation. Third, in general the marginal increase in profit also decreases as the amount of inventory increases. There are optimal inventory contents for individual outlets as well as the whole system for a given parameter setting of the problem. Fourth, the allocation of inventory among outlets is an important factor towards the profit of the system. In general the flexibility in inventory allocation increases the expected profit.
In the future, we can improve on the present work in two aspects. First, for special price-demand functions, we can search for specific basis functions to approximate the cost-to-go functions. The successful implementation of this approach would lead to an analytical approach that possibly gives close-form expressions for the approximate optimal policies. More qualitative property of the approximate optimal policies can then be deduced. Second, by modifying the current framework, we can study the quick response mode of fast fashion in which customer preference from a short, pilot-run period is used to determine the styles selected for mass production. In our single-product, pre-production context, we can consider outlets with different customer preferences such that knowledge learnt from the first period guides us the shipment of goods in later periods.

References


