Optical Spatial Solitons and Their Interactions: Universality and Diversity

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Spatial solitons, beams that do not spread owing to diffraction when they propagate, have been demonstrated to exist by virtue of a variety of nonlinear self-trapping mechanisms. Despite the diversity of these mechanisms, many of the features of soliton interactions and collisions are universal. Spatial solitons exhibit a richness of phenomena not found with temporal solitons in fibers, including effects such as fusion, fission, annihilation, and stable orbiting in three dimensions. Here the current state of knowledge on spatial soliton interactions is reviewed.

Optical spatial solitons (*1*) are self-trapped optical beams of finite spatial cross section that travel without the divergence associated with freely diffracting beams. This is possible only if the medium has a response that acts like a self-focusing mechanism for the light. The self-trapping is a consequence of a strong interaction between the medium and the electromagnetic wave, with the wave modifying locally the medium and being in turn modified by it. Hence, any beam whose field overlaps this perturbed region of space is affected. This is how solitons interact with each other, and in fact with any electromagnetic field. It is now known that self-trapping can exist by virtue of many different physical mechanisms, yet these solitons have many properties in common. Here we present the basic conceptual ideas involving optical spatial solitons, briefly discuss several mechanisms that support such solitons, and highlight the universality of the interactions between them that exists in spite of the widely diverse physical origins for the self-trapping (*1*).

When very narrow optical beams propagate without affecting the properties of a medium, they undergo natural diffraction and broaden with distance. The narrower the initial beam is, the faster it diverges (diffracts). In nonlinear materials, the presence of light modifies their properties (refractive index, absorption, or conversion to other frequencies). The refractive index change resembles the intensity profile of the beam, forming an optical lens that increases the index in the beam's center while leaving it unchanged in the beam's tails. This induced lens focuses the beam (Fig. 1A), a phenomenon called self-focusing that is a precursor of solitons. When self-focusing exactly balances beam divergence (diffraction, Fig. 1B), the beam becomes self-trapped at a very narrow width and is called an optical spatial soliton (Fig. 1C).

Solitons have been predicted and demonstrated in many physical systems: surface waves in shallow water (*2*), plasma waves (*3*), sound waves in ^{3}He (*4*), short temporal optical pulses in fibers (*5*), and optical spatial solitons. In spite of this diversity, interactions (collisions) between solitons in all of these systems follow the same principles. In the last few years, optical spatial solitons have become the main arena for studying soliton interactions and they are responsible for much of the recent conceptual progress on soliton phenomena, because of the ease with which sophisticated experiments can be conducted in a laboratory environment that offers precise control over almost every parameter. Furthermore, the ability to sample the waves directly as they propagate, and the availability of numerous material systems that are fully characterized by a set of simple equations result in a field in which theory and experiments make rapid progress hand-in-hand. The universal principle unifying all solitons is that the wave-packet (beam or pulse) creates, by virtue of the nonlinearity, a potential well and captures itself in it. It becomes a bound state of its own induced potential well. Thus, interactions between solitons are just interactions between bound states of a jointly induced potential well, or between bound states of different wells located at close proximity.

The Diversity of Solitons

We now briefly review the various kinds of optical spatial solitons that have been demonstrated experimentally (*6*). The first spatial solitons were suggested in nonlinear optical Kerr media in the 1960s (*7*). Kerr nonlinearities are characterized by a local, instantaneous refractive index change, $\Delta n = n_2 I$, where I is the local intensity and n_2 is a real constant. All media exhibit the optical Kerr effect at frequencies very far from any resonances so that the nonlinearity is very weak. Typical values of Δn are of the order of 10^{-4} or smaller. It became quickly clear that bright Kerr solitons are stable only in planar (1 + 1)D systems; bright (2 + 1)D solitons undergo catastrophic collapse (*8*), and (1 + 1)D soliton-like fields in a bulk [three-dimensional (3D)] medium suffer from transverse instabilities that break the beam apart (*9*, *10*). [The nomenclature (m + 1)D means that the beam can diffract in m dimensions as it propagates in one dimension.] Thus, observation of spatial Kerr solitons required slab waveguides, and it was not until the mid-1980s that such solitons were first observed (*11*). For $n_2 > 0$ (self-focusing), the soliton is robust against perturbations in width and intensity, as demonstrated theoretically and experimentally. Thus far, Kerr solitons have been observed in CS_2 (*11*), glass (*12*), semiconductor (*13*), and polymer waveguides (*14*). In bulk (3D) media, although there is a critical power for which self-focusing balances diffraction, any fluctuations in intensity or beam shape lead to either catastrophic self-focusing (and usually material damage) or to beam spreading (*8*).

The number of mechanisms that lead to effective self-focusing are more plentiful than would have been imagined 10 years ago. Most of them include some form of saturation of the nonlinear change in the refractive index, Δn, which allows stable self-trapping in bulk (3D) media. Following the classical mathematics nomenclature, these waves were originally called "solitary waves" to distinguish them from the Kerr-based "solitons." Modern nonlinear optics nomenclature now identifies all self-trapped optical beams as solitons (*1*). In 1974 Bjorkholm and Ashkin of Bell Labs were the first to demonstrate spatial solitons in bulk media, specifically in atomic (sodium) vapor (*15*). Recently, bulk spatial solitons were observed in semicon-

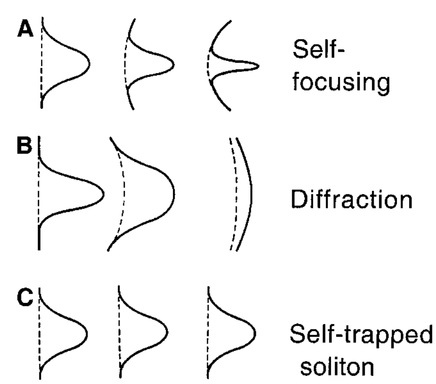

Fig. 1. Schematic showing the spatial beam profiles (solid line) and phase fronts (dashed line) for (**A**) beam self-focusing, (**B**) normal beam diffraction, and (**C**) soliton propagation.

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ductor gain media (*16*) and polymers (*17*).

Another class of solitons that relies on the index changes produced in photorefractive materials was discovered in the early 1990s (*18*, *19*). Photorefractive media sometimes contain sufficient scattering centers so that beam propagation can be seen visually: An image of beam diffraction with the nonlinearity off and soliton propagation with the nonlinearity on is shown in Fig. 2. The richness of nonlinear phenomena that exist in photorefractive materials gives rise to a family of solitons that rely on a diverse collection of nonlinear forms of the intensity-dependent index changes (*20*). Even in the simplest case of screening solitons (*21–24*), there are multiple physical effects involved. The net effect of the complex physics in all photorefractive solitons is that the underlying nonlinearities are saturable and the solitons are stable in both slab waveguides (*24*) and in bulk media (*19*, *20*, *23*).

Yet a third class of coherent spatial solitons, quadratic solitons, consisting of multifrequency waves coupled by means of a second order nonlinearity, was predicted in the mid-1970s (*25*) and realized experimentally in the 1990s (*26*). Here self-trapping occurs as a result of the rapid exchange of energy between the multifrequency waves, which keeps the power and spatial width of the beams mutually stabilized. This effect resembles a saturating nonlinearity, and the solitons are stable in both waveguides and bulk media (*26*, *27*).

For the cases just summarized, the nonlinear response of the medium was essentially instantaneous to changes in the optical field parameters. In practice, some nonlinearities have a rather slow response time (for example, photorefractive and thermal) (*28*). When the nonlinear response of the medium is slower than random fluctuations in the phase of the optical fields, it has proven possible to demonstrate incoherent solitons (*29*). The medium responds only to the time-averaged intensity of the optical fields (average taken over times much longer than the response time of the nonlinearity). Incoherent solitons are self-trapped entities (wave-packets) within which the phase varies randomly across the beam. They were first demonstrated in photorefractives, by using a phase diffuser that rotated faster than the nonlinear response of the medium (*29*), and later extended to the self-trapping of incoherent white light (from an incandescent light bulb) (*30*).

Despite the diversity of mechanisms responsible for self-trapping, all spatial solitons have multiple properties in common. The fields die off in an evanescent manner. Other properties, such as phase velocity, nonlinear phase shift, and spatial width, depend on their peak power. However, perhaps their most fascinating feature is their mutual interaction. In many aspects, solitons interact like particles despite the electromagnetic wave nature of their fields. The existence of the interaction is manifested in a change in their trajectories when a second soliton is introduced so that the soliton fields overlap significantly in their tails. As the soliton fields by definition modify the optical properties of the medium, the propagation characteristics of any other electromagnetic wave, a second soliton, are also affected.

Composite (Vector) Solitons

Composite (vector) solitons consist of two (or more) components (modes) that mutually self-trap in a nonlinear medium (*31*). A key prerequisite for forming a stationary vector soliton is that the interference between the modes does not contribute to the nonlinear index change, Δn. Otherwise the induced waveguide does not have a constant shape (in the propagation direction) and thus the components self-trapped within it are not stationary (*32*). Vector solitons, first suggested by Manakov (*33*), consist of two orthogonally polarized components in a nonlinear Kerr medium in which self-phase modulation is identical to cross-phase modulation (*34*). That is, the nonlinear action of a field component on itself equals the action of one component on the other. Under these assumptions, Manakov has proven that his vector solitons form an integrable system, in which they interact as fully elastic collisions, conserving power, linear momentum, and the number of solitons. This requirement of equal self- and cross-phase modulation occurs in very few materials. Furthermore, even though orthogonal polarizations eliminate interference between the field components, another nonlinear term [called the four-wave-mixing term (FWM)] exists in all optical $\chi^{(3)}$ media and does not allow ideal Manakov solitons to exist. Fortunately, both an appropriate nonlinear material and appropriate experimental conditions for minimizing FWM were found and Manakov solitons were demonstrated (*35*).

Two other techniques for eliminating the contribution of interference to Δn (and thus for generating stationary vector solitons) were proposed. In the first, each field component is at a different frequency, and the frequency difference between components is much larger than τ^{-1}, where τ is the nonlinearity relaxation time (*36*, *37*). In the second, the field components are incoherent with one another (*38*). Stationary propagation of a vector soliton is achieved if the field components correspond to guided modes of the waveguide induced by the sum of their intensities. Once the interference contribution Δn is eliminated, there is no reason why the field components cannot have the same polarization, so the self- and cross-phase modulation can be easily made equal, which should facilitate future observations of Manakov solitons in noninstantaneous Kerr media.

The various vector soliton constituents just discussed have always populated the lowest (fundamental) mode of their jointly induced waveguide. However, vector (multicomponent) solitons can also consist of different modes of their jointly induced waveguide (*39*, *40*). The total intensity profile of these multimode solitons may possess multiple humps (*41*). They were demonstrated experimentally in photorefractive media and found to be stable (or weakly unstable) in large regions of their parameter space (*41*, *42*). Finally, composite solitons can sometimes form "hybrid" structures in which one component is "bright" and the other is "dark" (an intensity null) (*36*, *43*), as observed in (*37*, *44*).

Soliton Collisions: Basic Concepts

Interactions between solitons are perhaps the most fascinating features of soliton phenomena. There are two categories: coherent and incoherent interactions. Coherent interactions occur when the nonlinear medium can respond to interference effects that take place when the beams overlap. They occur for all nonlinearities with an instantaneous (or extremely fast) time response (the optical Kerr effect and the quadratic nonlinearity). Materials with a long response time (photorefractive and thermal) only respond to interference between the overlapping beams if the relative phase between the beams is stationary for a time much longer than τ. The solitons then exert attractive or repulsive forces on each other, depending on their relative phase (*45*). Incoherent interactions, on the other hand, occur when the relative phase between the (soliton) beams varies much faster than τ. In this case the resultant force between such bright solitons is always attractive (*46*).

Soliton interactions are sufficiently complex that it is frequently necessary to resort to detailed numerical calculations for predictions. However, a few cases can be analyzed using inverse scattering theory for the $(1 + 1)$D Kerr case (*47*). First, because Kerr solitons are $(1 + 1)$D, their collisions occur in a single plane. Second, all collisions are fully elastic so that the number of solitons is always conserved. Third, the system is integrable, and therefore no energy is lost (to radiation waves). Finally, the directions and propagation velocities of the solitons recover to their initial values after each collision. This equivalence between solitons and particles was first suggested in 1965 and led to the term "soliton" (*48*). The real surprise was, however, that solitons survive the collision event as self-trapped entities, even though the solitons themselves are highly nonlinear creatures. Furthermore, the collision between solitons involves "forces" (*45*): Solitons interact like real particles, exerting attraction and repulsion on one another.

The simplest collisions occur for two parallel launched equivalent solitons when their relative phase is zero (in-phase, $\Delta\phi = 0$). When the solitons interfere coherently, the intensity in the center region between the induced waveguides is increased (Fig. 3), which leads to an increase in the refractive index in that region for $n_2 > 0$. This in turn attracts more light to the center, moving the centroid of each soliton toward it, and hence the solitons appear to initially attract each other. Detailed analysis of the evolution shows that the force is indeed initially attractive and there is no energy exchange between the solitons. This feature is universal for all coherently interacting solitons in isotropic nonlinear media.

The behavior subsequent to the first merging of the solitons depends on whether the nonlinear response is pure Kerr, or saturating. For two equivalent Kerr solitons on initially parallel trajectories, the resulting path of the centroid of each individual soliton is periodic with the solitons returning to their input condition at the end of each cycle (Fig. 4A). For large enough divergent input angles, the solitons never collide. For large enough converging angles, the solitons "pass through" each other with a slight lateral deflection and thereafter diverge (Fig. 4B).

Interacting beams π out of phase from each other interfere destructively, and the index in the central region is lowered by their overlap (Fig. 3). Therefore, the centroid of each soliton moves outward and the solitons appear to repel each other (Figs. 3 and 4C).

The situation is more complex for other relative phases. If there were no power transfer between the solitons, the force between the solitons would vary smoothly from maximum attractive at $\Delta\phi = 0$ to maximum repulsive at $\Delta\phi = \pi$. However, there is a component to the interaction that varies approximately as $\sin\Delta\phi$ and leads to power transfer between the two solitons. That is, one soliton grows in net energy with respect to the other. The net energy transfer is reversed in the relative phase regions $0 \leq \Delta\phi \leq \pi/2$ and $\pi/2 \leq \Delta\phi \leq \pi$. As the amplitudes and relative phases of the solitons change with distance, their widths also change in keeping with the appropriate relation between width and peak power for Kerr solitons. Consequently, the details of the trajectories can be quite complex (Fig. 4, D and E).

The collision of nonequivalent coherent solitons always results in energy exchange and leads ultimately to a repulsive force that makes the beams diverge.

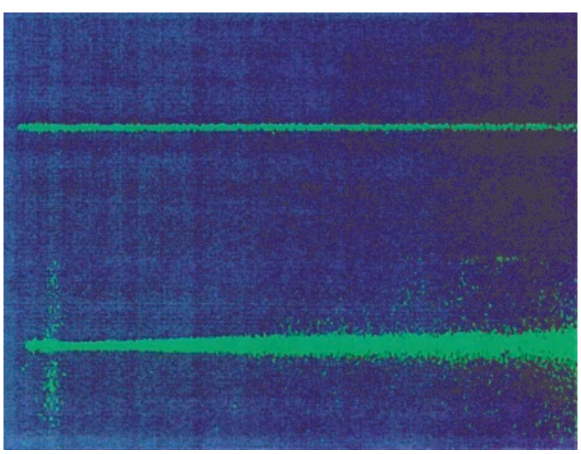
Collisions in saturable nonlinear media are more diverse and interesting than those found in Kerr media, because saturable nonlinear media can support (2 + 1)D solitons, allowing collisions in full 3D and new phenomena such as soliton fusion (*49*, *50*), fission (*50*), and annihilation to occur. Fusion (decrease in soliton number on collision) occurs for parallel input solitons (or small enough relative angles), when the collision angle is less than the maximum total internal reflection angle (TIRA) in the induced waveguide (*50*). In terms of a "potential well," capture depends on whether the kinetic energy of the colliding wave-packets results in a velocity that is smaller than the escape velocity. The solitons can fuse together either on the first merging (Fig. 4F) or after a finite number of oscillations of decreasing amplitude and period.

For collision angles larger than the TIRA, the solitons simply go through each other unaffected, and for incidence angles less than the TIRA, the beams can couple light into each other's induced waveguide. If the induced waveguide can guide only a single mode, the outcome will resemble a collision in Kerr media (with some small energy loss to radiation). However, for an induced multimode waveguide, higher modes can be excited, and in some cases the solitons fuse to form one soliton beam, accompanied by small energy loss to radiation, much like inelastic collisions between real particles (*50*). This naïve picture of soliton interactions gives qualitative understanding of soliton collisions. In reality, the interacting solitons affect each other's induced waveguide, and the true collision process is much more complicated. In summary, the new key features introduced by the saturating nature of the nonlinearity are full 3D interactions and the fact that the soliton number is not necessarily conserved.

Because quadratic solitons do not involve any real index changes, one might expect that their interactions could exhibit different features. Here the interaction involves the different frequencies that generate the solitons through the quadratic parametric process. Although the interactions of quadratic solitons are different in the details of the physics, they are similar to those obtained in other saturable nonlinear media (*51*). This highlights the universality of soliton phenomena that are largely independent of the actual physical mechanism that enables them.

Soliton Collisions: Experiments

Coherent collisions in Kerr slab waveguide media have been demonstrated in carbon disulfide (*52*), glass (*53*), and AlGaAs (*54*). The attraction and repulsion for $\Delta\phi = 0$ and π were



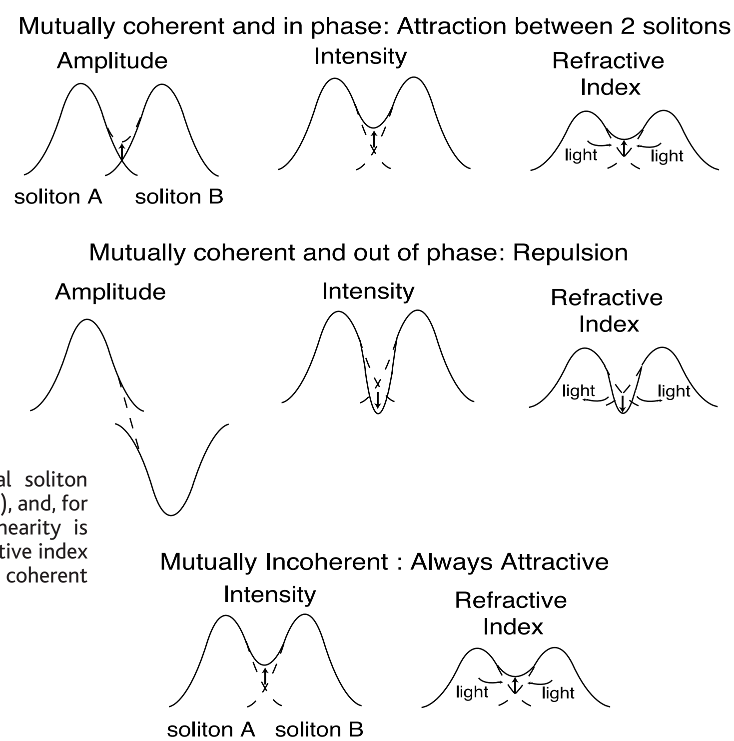


Fig. 2 (above). A top view photograph of a 10-μm-wide spatial soliton propagating in a strontium barium niobate photorefractive crystal (top), and, for comparison, the same beam diffracting naturally when the nonlinearity is "turned off" (bottom). (*23*). **Fig. 3 (right).** Schematic of the refractive index spatial distribution for a collision between in-phase and out-of-phase coherent spatial solitons.

clearly observed, as was the exchange of soliton power, which is maximized at $\Delta\phi = \pi/2$ and $3\pi/2$, and the reversal in power transfer direction due to change in $\Delta\phi$ of π.

Similar effects have been observed in media with saturating nonlinearities, for example, with quadratic solitons (*55*) (Fig. 5). Many phenomena associated with saturating nonlinearities have been illustrated with photorefractive solitons because of the ease of generating them and observing their detailed trajectories. Experimentally, fusion of solitons has been observed in all kinds of saturable nonlinear media: atomic vapor (*56*), photorefractives (*57*, *58*), and quadratic (*55*). Fission (breakup) has also been observed with photorefractive solitons (*57*) and predicted to occur with quadratic solitons as well (*59*). Annihilation of solitons upon collision occurred when three solitons collided and only two emerged from the collision process (*60*).

Incoherent collisions occur when the relative phase between the solitons varies much faster than τ. All of the existing research has dealt with photorefractive solitons (*61–64*). Because the medium responds only to the time-averaged intensity, the intensity in the center region between the solitons is increased (Fig. 3), which increases the refractive index in that region. Thus, light is attracted toward the center region, and the solitons attract each other. As a result, solitons either pass through each other, fuse, or break up into additional solitons during a collision, as discussed previously for the saturating nonlinearity case.

3D Soliton Interactions

Because saturable nonlinear media can support (2 + 1)D solitons, collisions can occur with 3D trajectories. When nonparallel solitons are launched in different planes, they interact (attract or repel) and their trajectories bend. Such a system possesses initial angular momentum. If the soliton attraction exactly balances the "centrifugal force" due to rotation, the solitons can capture each other into orbit and spiral about each other, much like two celestial objects or two moving charged particles do. This idea, first suggested for coherent solitons (*65*), was found to lead to spiraling-fusion and spiraling-repulsion (*56*) but not stable orbits in a saturable self-focusing medium because the interaction was coherent and critically sensitive to phase. Similar results were obtained (*66*) with quadratic solitons, and a theoretical study of the spiraling possibility of such solitons predicted "bouncing" off each other and no stable orbits either (*67*). The centrifugal force between solitons is always "repulsive," so a spiraling orbit requires soliton attraction, which is only obtained for coherent solitons when they are exactly in phase and identical. However, any tiny perturbation in the phase or amplitude of one of the solitons results in power exchange and ultimately leads to the solitons propagating at slightly different velocities, thus increasing the relative phase that unbalances the forces. Realizing that the force between incoherent solitons is purely attractive, and is independent of the relative phase, has led to the observation (Fig. 6) of spiraling-orbiting solitons in an elliptical orbit (*62*, *63*). When the initial distance between the solitons is increased, the solitons do not form a "bound pair." On the other hand, when their separation is too small, they spiral on a converging orbit and eventually fuse.

Recent theoretical and experimental work has shown that the interaction mechanism of spiraling solitons is richer than initially thought (*63*). The two spiraling-interacting solitons exchange energy by coupling light into each other's induced waveguide, a consequence of a saturable nonlinearity and trajectories at a small relative angle. But, because the two interacting solitons have equal power, the energy exchange is symmetric. The energy exchanged is phase-coherent with its source but phase-incoherent with the soliton into which it was coupled. Thus, the energy exchange induces partial coherence between the initially incoherent solitons and hence affects the forces involved. The result is that the two solitons orbit periodically about each other and at the same time exchange energy periodically, with the two periods (the spiraling period and the energy exchange period) being only indirectly related. The complicated motion is stable over a wide range of parameters (*63*). This 3D bound state of spiraling solitons conserves angular momentum. Recalling that saturable nonlinearities lead to nonintegrable equations, it is surprising that the complicated dynamics does not lead to measurable escape of energy to radiation and to the destruction of this 3D bound state of solitons. Whether or not the spiraling can continue indefinitely

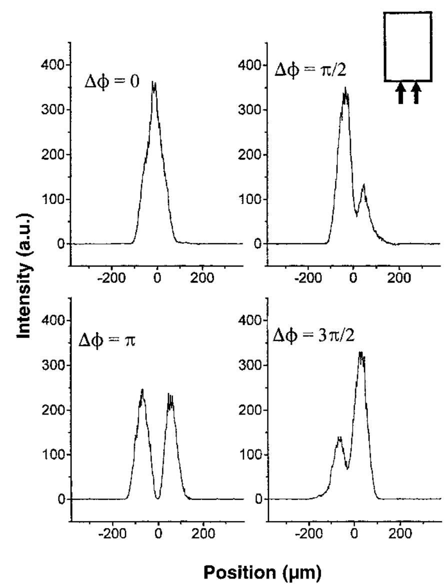


Fig. 5. The experimentally measured output from the interaction between two parallel quadratic solitons (saturating nonlinearities) for different relative input phases $\Delta\phi$ between the fundamental beams (*56*).

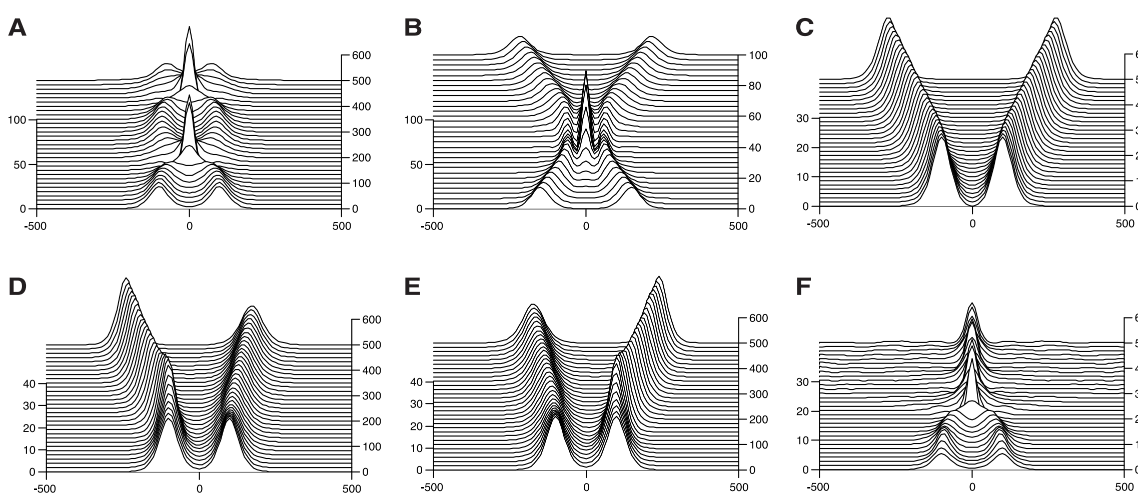


Fig. 4. Beam evolution calculations of the interactions between two solitons for the following cases: (**A**) Parallel input trajectories, in-phase Kerr solitons; (**B**) converging input trajectories, in-phase Kerr solitons; (**C**) parallel input trajectories, out-of-phase Kerr solitons; (**D**) parallel input trajectories, $\pi/2$ relative phase between Kerr solitons; (**E**) parallel input trajectories, $3\pi/2$ relative phase between Kerr solitons; and (**F**) fusion of two solitons input on parallel trajectories in saturating nonlinear media for "small" input separations.

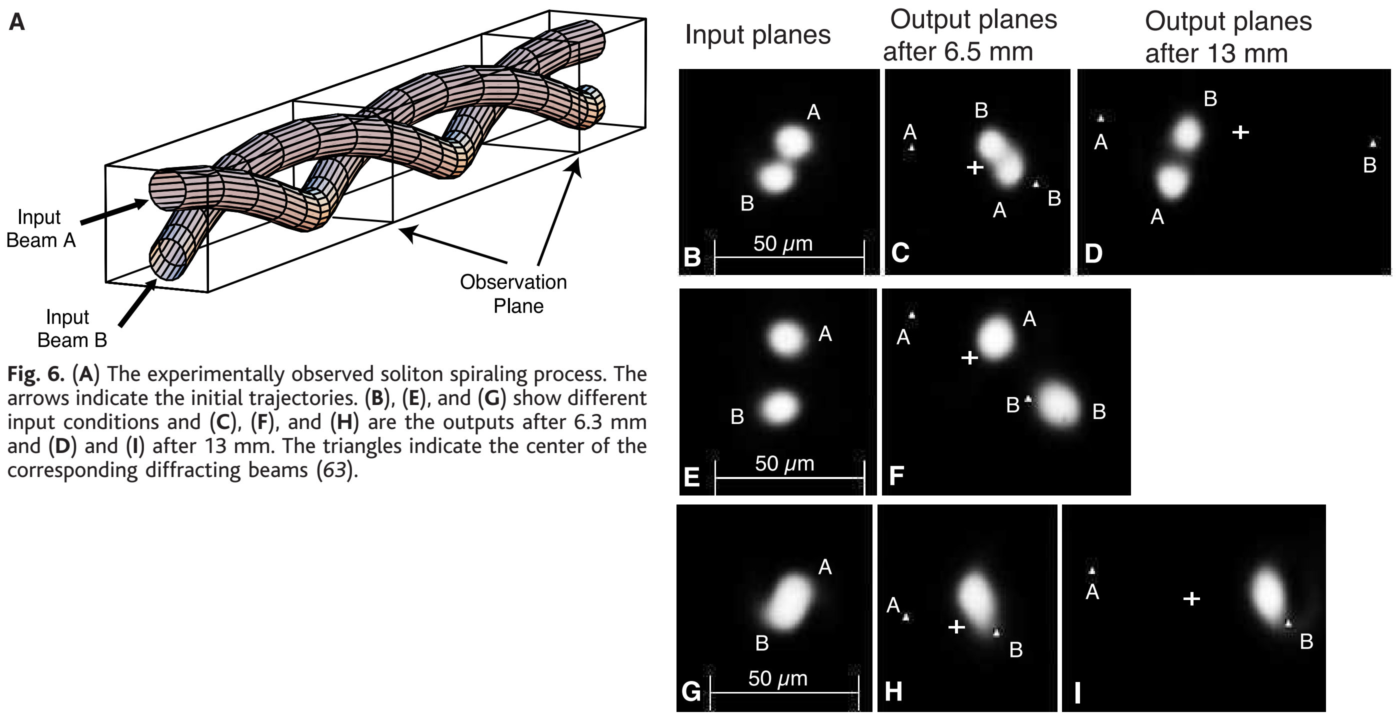


Fig. 6. (**A**) The experimentally observed soliton spiraling process. The arrows indicate the initial trajectories. (**B**), (**E**), and (**G**) show different input conditions and (**C**), (**F**), and (**H**) are the outputs after 6.3 mm and (**D**) and (**I**) after 13 mm. The triangles indicate the center of the corresponding diffracting beams (*63*).

remains an open question, because it is possible that for long propagation distances (well beyond present experimental feasibility), the solitons eventually merge. Finally, when a tiny seed in one of the input solitons is coherent with the other soliton, the relative phase between these coherent components controls the outcome of the collision process and can turn a spiraling motion into fusion or repulsion (*63*).

Interactions Between Composite Solitons

Collisions between vector (composite) solitons have some unique features. Shape transformations of colliding multimode solitons (*68*) can lead to two different multimode solitons emerging from the collision process. Another example is energy exchange between the components of colliding vector solitons without radiative losses (*69*), which was predicted initially for Manakov solitons and named "polarization switching" (*70*). Finally, a bound state between two vector solitons, each being a dark-bright soliton pair, was demonstrated (*71*). That experiment revealed some similarity (albeit incomplete) to gluons from quantum chromodynamics. Recently, composite (multimode multihump) solitons were also suggested in (2 + 1)D with the exciting options of carrying topological charge in one of the vector components (*72*), or in the case of three or more vector constituents, carrying different charges on different components. Unlike the case of dark vortex solitons (*73*), the topological charge is carried by a soliton of finite energy and therefore plays the role of spin in real particles. It is obvious that the "spin" (topological charge) carried by the various constituents, the multimode nature and the multihump structure, offer many new exciting possibilities for interactions between 2D vector solitons. It is possible that collisions between such composite (2 + 1)D solitons could conserve not only energy, and linear and angular momentum, but also the equivalent of spin.

Summary

Collisions between spatial solitons exhibit many interesting and diverse outputs. The largest variety of phenomena occur for a combination of saturable media and multicomponent composite (vector) solitons. Independent of the exact physical origin of the saturating nonlinearity, the phenomena are similar and exhibit universal properties. Hence photorefractive solitons, quadratic solitons, and solitons in media whose index change saturates with increasing intensity all exhibit the same interactions. In fact, interactions between any Kerr solitons also form a universal class, similar to, but reduced somewhat relative to, that of saturating nonlinearities. This similarity in interactions is likewise true for vector solitons, whose features are characterized by the composite structure, irrespective of the physical mechanism that gives rise to self-trapping. The universality manifested in the collision properties of solitons is one of their most appealing aspects. It is already clear that self-trapped wave-packets (solitons) and the de Broglie wave representation of real particles have many properties in common. Are there fundamental new laws of physics linking solitons and particles? We believe that indeed this is the case, and conjecture that in the next decade many of them will be discovered.

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74. This research was supported at Princeton by the U.S. Army Research Office and the National Science Foundation, at the Technion by the Israel Science Foundation and by the Israeli Ministry of Science, and at CREOL by the National Science Foundation.

REVIEW

Nonlinear Optics for High-Speed Digital Information Processing

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Recent advances in developing nonlinear optical techniques for processing serial digital information at high speed are reviewed. The field has been transformed by the advent of semiconductor nonlinear devices capable of operation at 100 gigabits per second and higher, well beyond the current speed limits of commercial electronics. These devices are expected to become important in future high-capacity communications networks by allowing digital regeneration and other processing functions to be performed on data signals "on the fly" in the optical domain.

Nonlinear optical effects are not part of our everyday experience. At the relatively low light intensities that normally occur in nature, the optical properties of materials at any instant in time are independent of the intensity of illumination. When light waves pass through a medium, there is no interaction between the waves. These are the properties of matter that are familiar to us through our visual sense. However, if the illumination is made sufficiently intense, the optical properties of the medium begin to depend on the intensity and other characteristics of the light. For example, the refractive index n of the medium is changed by an amount $\Delta n = n_2 I$, where I is the optical intensity and n_2 is the nonlinear refraction coefficient. The incident light waves may then interact with each other as well as with the medium. This is the realm of nonlinear optics (*1, 2*).

Within the past decade, optical fiber cable has been installed in vast quantities in telecommunications networks throughout the world, and the use of light for transmission of information has become ubiquitous. Already,

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