Phase-dependent collisions of (2+1)-dimensional spatial solitons

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Coherent collisions of (2 + 1)-dimensional spatial solitons in photorefractive media are studied. Phase-dependent switching of the spatial location of the output beams is demonstrated. The experimental results are in close agreement with a three-dimensional numerical calculation. © 1998 Optical Society of America [S0740-3224(98)03507-3]

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1. INTRODUCTION

The possibility of using spatial solitary waves for implementation of optical logic and switching has led to a number of experimental and theoretical investigations of the interactions between spatial solitons.[1–8] An extensive discussion of geometrical and material alternatives for implementing optical soliton switches was given in Ref. 9. Initial experimental work in this area centered on Kerr or saturable Kerr-type nonlinearities in which phase-dependent attraction and repulsion of one-dimensional, planar soliton beams was observed in media with a self-focusing nonlinearity.[2,3] Related studies of dark soliton dynamics in self-defocusing media have also been reported.[7]

During the past few years steady-state self-focusing and formation of spatial solitons in photorefractive media have been reported by several groups.[10–12] Interest in photorefractives in the context of spatial switching stems in part from the possibility of soliton propagation at relatively low levels of optical power. Self-focusing effects, and convergence to solitary profiles, have been observed for beams at the microwatt-power level, which is significantly lower than that required in traditional Kerr-type media. The interaction of both mutually coherent[13–15] and mutually incoherent[16,17] solitons has been studied. Mutually coherent beams exhibit attractive or repulsive forces, depending on their relative phase. For particular initial conditions excitation of a higher-order bound dipole state consisting of a pair of beams with π relative phase shift has been demonstrated.[18] The interaction of incoherent beams in self-focusing-type media is qualitatively different from that in the coherent case. In the former the relative phase plays no role, and the addition of intensities leads to an attractive force between the solitons.

In this paper the interaction of mutually coherent two-transverse dimensional {(2 + 1)-dimensional [(2 + 1)D]} bright spatial solitons in a photorefractive crystal is investigated. We show that switching the relative phase of the two input beams leads to a well-resolved spatial shift of the beams at the output of the crystal. The interaction thus implements a phase-dependent optical switch. When the input beams are in phase and have a sufficiently small crossing angle, the output beams are observed to fuse. This behavior was also reported in Ref. 14, and may be understood if we recall that we are dealing with a nonintegrable system. A cubic Kerr nonlinearity in one-transverse dimension is an integrable model, within which soliton collisions are purely elastic.[19] Here we work with a nonintegrable system in which inelastic scattering between solitary waves occurs.

Differences between the photorefractive and Kerr or saturable Kerr nonlinearity lead to some specific features of the soliton interactions studied here. In contrast to Kerr-type media, in which frequency-degenerate temporally stationary beams couple via a $\chi^{(3)}$ coefficient that is purely real, the coupling constant γ in photorefractive media is generally complex. The real part of γ, which is of interest here, leads to self-focusing, soliton formation, and phase coupling between interacting beams. The imaginary part of γ leads to stimulated photorefractive scattering, also known as fanning,[20] and to energy coupling between interacting beams. In the context of soliton studies in photorefractive media the fanning results in radiation losses and in bending of the solitary profile.[21,22] As described in Section 2, the choice of a specific interaction geometry minimizes the influence of the imaginary part of γ.

2. EXPERIMENTS

The experiments were performed in a photorefractive crystal of strontium barium niobate (SBN), which has a polar symmetry axis (c axis). Crossing beams transfer energy such that the beam whose $\mathbf{k}$ vector makes a smaller angle with the c axis is amplified. Unidirectional energy transfer may be undesirable in an optical switching fabric. To minimize the energy transfer we use the interaction geometry shown in Fig. 1, in which both beams lie in the plane perpendicular to the c axis. Even though the energy coupling is minimized by this choice of geometry, we effectively retain a strong nonlinear coupling because of the real part of γ. We have confirmed by numerical simulation that in this interaction geometry the results obtained in photorefractive media are qualitatively similar to those obtained in a material with isotropic Kerr nonlinearity. One noticeable difference is the elliptical shape of the solitons, which is due to the anisotropy of the photorefractive response.[12]

Copropagating parallel beams experience phase-dependent forces such that in-phase beams attract each other, while beams with a π phase shift repel each other. We consider here the case of colliding beams that cross at the input face of the nonlinear medium. In-phase beams give a bright fringe at the input face and interact strongly. The beams attract each other, and when the angle between them is not too large they merge, as is the case for incoherent collisions at a small angle.[17] Beams with a π mutual phase shift give a central dark fringe. When the crossing angle is comparable with the divergence angle of each input beam there is little overlap between the beams in the nonlinear medium, and they interact weakly. The weak interaction is a linear effect due to the choice of interaction geometry; in general, the nonlinear attractive and repulsive forces are equally strong.

Experiments were performed with a 10-mW He–Ne laser beam ($\lambda = 0.63\ \mu$m), which was passed through a system of lenses controlling the size of the beam waist, and a beam splitter arrangement used to create two beams. Each beam had a power of approximately 40 μW. The beams had a variable crossing angle and adjustable relative phase. The beams were directed into a photorefractive crystal of SBN:60 doped with 0.002% by weight Ce. The measured linear absorption coefficient of the crystal, at the wavelength used, was rather small, approximately 0.02 cm^{-1}, and hence played no role in the reported results. The axial position of the crystal was adjusted so that the beams intersected at the input face of the crystal. The beams propagated perpendicular to the crystal $\hat{c}$ axis ($=z$ axis) and were polarized along it to take advantage of the largest component of the electro-optic tensor of SBN, r_{33}, which was measured to have the value of 120 pm/V. The crystal measured 20 mm along the direction of propagation and was 5 mm wide along the $\hat{c}$ axis. The nonlinearity was controlled by application of a dc voltage along the $\hat{c}$ axis, and the effective saturation intensity was controlled by illumination of the crystal from above with incoherent white light. The experiments were performed with $I_m \simeq 0.5$, where I_m is the peak intensity of each beam normalized to the effective saturation intensity. The value of I_m was estimated by measurement of two-beam coupling gains with and without the incoherent illumination, as explained in Ref. 12. In this regime there is monotonic self-focusing of initially round input beams and convergence to elliptically shaped solitary profiles that are squeezed along the direction of the $\hat{c}$ axis.[12] Images of the beams at the input and output faces of the crystal were recorded with a CCD camera.

Figure 2 shows the result of soliton collisions for 0 (row $A + B$) and π (row $A - B$) relative phase shift between the beams. All the pictures were recorded in steady state after the decay of any transients. The top row (A, B) depicts the result of propagating the beams independently through the medium (the resulting intensity pictures were added to create the frames in the figure). The inputbeams had a full width at half-maximum (FWHM) diameter of 31 μm, while the diffracted output beams for

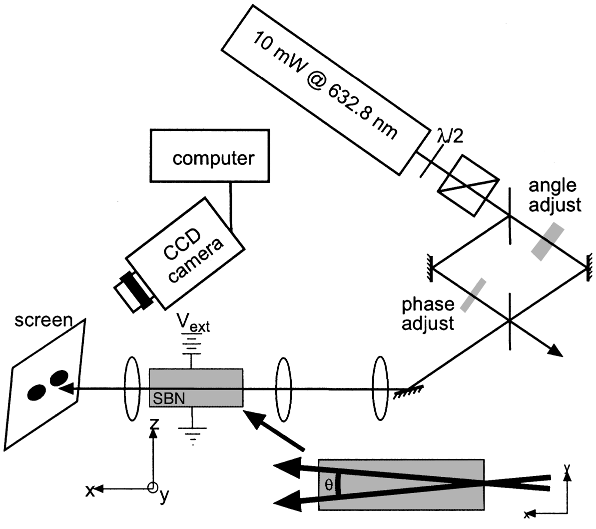

Fig. 1. Experimental setup.

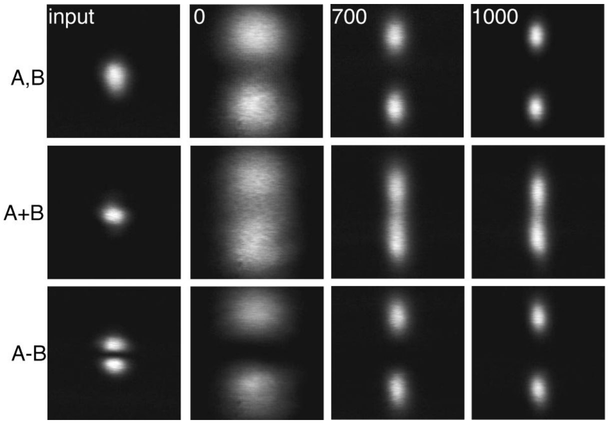

Fig. 2. Phase-dependent soliton collisions with internal crossing angle $\theta = 6.4$ mrad. The leftmost column shows the input beams. Succeeding columns show output beams at the indicated value of the external voltage. Each frame depicts a 235 μm × 235 μm region, and the z axis is horizontal.

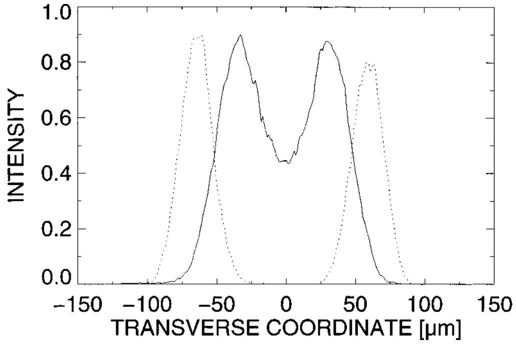

Fig. 3. Intensity distributions along the y axis for $A + B$ (solid curve) and $A - B$ (dashed curve) interactions. The curves represent an integral over the z dependence of the intensity.

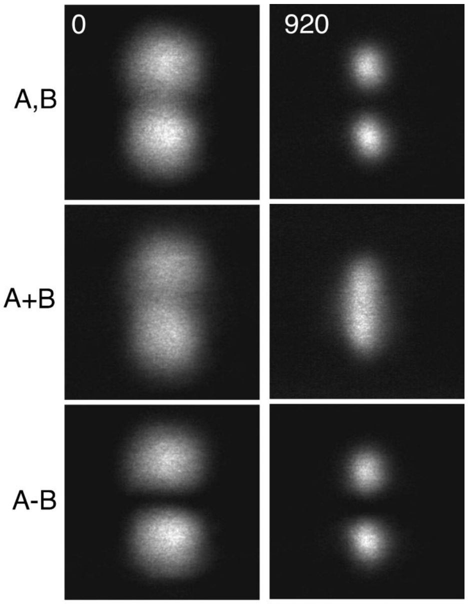

Fig. 4. Output beams for 0 (left-hand column) and 920 V (right-hand column), for internal crossing angle $\theta = 5.7$ mrad. Each frame depicts a 300 μm × 300 μm region, and the z axis is horizontal.

zero applied voltage had a FWHM diameter of 81 μm. As the voltage is increased, the beams self-focus and assume an elliptical soliton shape with $\mathrm{FWHM}_z = 23\ \mu\mathrm{m}$ and $\mathrm{FWHM}_y = 34\ \mu\mathrm{m}$ at 1000 V. In-phase inputs attract, and there is partial coalescence of the output beams. In the out-of-phase case the beams interact only weakly, as discussed above, and the output beam locations are close to those in the absence of interaction (Fig. 2, top row).

The intensity profiles along y of the output beams observed at 1350 V are shown in Fig. 3. A π phase shift between the input beams results in displacement of the output beams by slightly more than the FWHM diameter. This demonstrates the possibility of a high-contrast phase-dependent switch.

The degree of fusion observed for in-phase inputs depends sensitively on the crossing angle between the beams. The results obtained when the crossing angle is reduced to $\theta = 5.7$ mrad are shown in Fig. 4. In this case θ is more than 10% less than the characteristic full diffraction angle of the input beams, which was approximately 6.6 mrad, and there is strong attraction between in-phase inputs. This can be seen in the middle row of Fig. 4, in which the output beams at 920 V exhibit strong coalescence. Additional data taken with a crossing angle of approximately 7 mrad show that in-phase beams pass through each other without fusing.

3. SIMULATIONS AND DISCUSSION

We simulated the observed behavior by using the set of equations[23]

$$\left(\frac{\partial}{\partial x} - \frac{i}{2}\nabla^2\right)B(\mathbf{r}) = i\,\frac{\partial\varphi}{\partial z}\,B(\mathbf{r}), \tag{1a}$$

$$\nabla^2\varphi + \nabla\ln(1 + |B|^2)\cdot\nabla\varphi = \frac{\partial}{\partial z}\ln(1 + |B|^2), \tag{1b}$$

which describe time-independent propagation along the coordinate x in the presence of an electric field applied along z. Here $B(\mathbf{r})$ is the amplitude of the optical beam, $\nabla = \hat{y}(\partial/\partial y) + \hat{z}(\partial/\partial z)$ is the transverse gradient, and φ is the electrostatic potential induced by the beam with the boundary conditions $\nabla\varphi(\mathbf{r} \to \infty) \to 0$. Equations (1) are written in dimensionless coordinates. The electromagnetic intensity is normalized to the characteristic saturation intensity so that the saturation intensity in Eq. (1b) is equal to unity. More details about the derivation of Eqs. (1) and the normalizations used can be found in Refs. 23 and 24. Theoretical analysis and experimental observations of single and dipole soliton solutions of Eqs. (1) were discussed in Refs. 12 and 18, respectively (see also Ref. 25 for an overview).

Calculated output profiles are shown in Fig. 5 for parameters corresponding to Fig. 2. Numerical runs per-

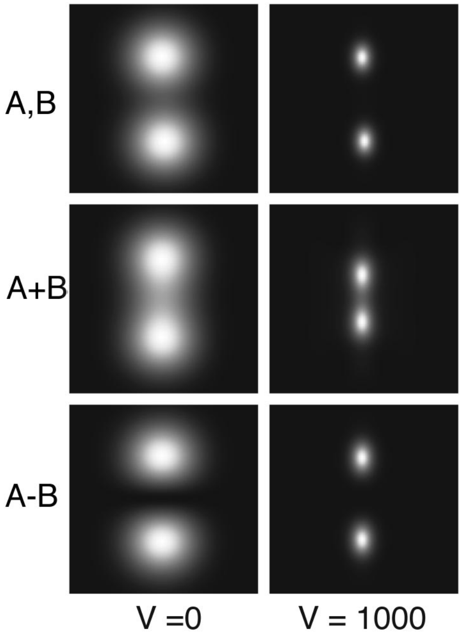

Fig. 5. Numerical results showing output intensity distributions for $A + B$ and $A - B$ interactions for an external field of 2000 V/cm and $\theta = 6.4$ mrad.

formed for longer crystals and/or higher voltages show complete merging of the input beams in the $A + B$ case at the given value of θ. Calculations for increasing values of nonlinearity show a slight attraction of the output beams in the $A - B$ case, as can be seen in the last column of Fig. 2. Calculations were also performed with an isotropic saturable Kerr nonlinearity $[\partial\varphi/\partial z \sim |B|^2/(1 + |B|^2)$ in Eq. (1a)]. The qualitative dependence of the results on the relative phase of the input beams was the same as that shown in Fig. 5.

In summary, we have investigated coherent interactions of colliding $(2 + 1)$D spatial solitons. Equal phase input beams attract each other, while out of phase input beams repel each other. When the crossing angle is smaller than approximately the characteristic diffraction angle the beams fuse, whereas at larger crossing angles they remain separated. Switching the relative input phase by π leads to high-contrast spatial switching of the outputs. While this work demonstrates the feasibility of controlling light by light in a spatial optical switch, the experimental parameters are not entirely satisfactory from the point of view of applications. Use of a photorefractive crystal allows switching to be achieved at low (μW) power levels, but the switching speed is correspondingly slow (of the order of seconds).

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