Spatial solitons in centrosymmetric photorefractive media

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We predict spatial solitons in photorefractive centrosymmetric media driven by the dc Kerr effect.

The richness of the nonlinear-optical effects in photorefractive media has given rise to a great deal of new soliton[1] phenomena in these materials. Since the initial discovery of spatial solitons in photorefractive media,[2] several distinctly different types of self-trapping effects have been predicted and observed. Thus far quasi-steady-state solitons,[2,3] photovoltaic solitons,[4,5] screening solitons,[6–10] resonant self-trapping in photorefractive semiconductors,[11] and self-trapping of spatially incoherent beams[12,13] and of incoherent white-light beams have been investigated.[14] In all these studies the nonlinear change in the refractive index resulted from the electro-optic (Pockels) effect and, as such, can exist only in noncentrosymmetric media. In general, however, the photorefractive effect also exists in centrosymmetric media and is driven by the dc Kerr effect.[15,16] Here we predict spatial solitons in photorefractive centrosymmetric media.

We start with the set of rate and continuity equations and Gauss's law, which describe the photorefractive effect in a medium in which electrons are the only charge carriers, plus the wave equation for the slowly varying amplitude of the optical field. In steady state and in two dimensions these equations are

$$(s|A|^2 + sI_b + \beta)(N_d - N_d{}^i) - \gamma \hat{n} N_d{}^i = 0\,, \tag{1}$$

$$\nabla \cdot \hat{\mathbf{J}} = \nabla \cdot (q\mu \hat{n}\hat{E} + k_B T \mu \nabla \hat{n}) = 0\,, \tag{2}$$

$$\nabla \cdot \hat{\mathbf{E}} + (q/\epsilon_s)(\hat{n} + N_A - N_d{}^i) = 0\,, \tag{3}$$

$$\left(\frac{\partial}{\partial z} - \frac{i}{2k}\frac{\partial^2}{\partial x^2}\right) A(x,z) = \frac{ik}{n_b}\Delta n(\hat{\mathbf{E}}) A(x,z)\,, \tag{4}$$

$$V = -\int_{-l/2}^{l/2} \mathrm{d}x \cdot \hat{\mathbf{E}}\,, \tag{5}$$

where $\Delta n(\hat{\mathbf{E}}) = {}^1\!/_2 n_b{}^3 g_{\mathrm{eff}} \epsilon_0{}^2 (\epsilon_r - 1)^2 \hat{\mathbf{E}}^2$ is the change in the refractive index,[15,16] assuming that the induced dc polarization is in the linear region, i.e., $\mathbf{P} = \epsilon_0(\epsilon_r - 1)\hat{\mathbf{E}}$. Note that Δn is now proportional to the space-charge field squared ($\hat{\mathbf{E}}^2$) rather than to $\hat{\mathbf{E}}$ as for all other photorefractive solitons driven by the Pockels effect. The independent variables are z, the propagation axis, and x, the transverse coordinate. The dependent variables are as follows: $\hat{n}$ is the electron density, $N_d{}^i$ is the density of ionized donors, $\hat{\mathbf{J}}$ is the current density, $\hat{\mathbf{E}}$ is the space-charge field inside the crystal, and A is the slowly varying amplitude of the optical field defined by $E_{\mathrm{opt}}(x,z,t) = A(x,z)\exp(ikz - i\omega t) + \mathrm{c.c.}$ ($k = 2\pi n_b/\lambda$, ω is the frequency, and n_b is the unperturbed refractive index). Relevant parameters of the crystal are as follows: N_d is the total donor number density, N_A is the density of negatively charged acceptors, β is the dark generation rate, s is the photoionization cross section, γ is the recombination rate, μ is the electron mobility, $\epsilon_s = \epsilon_r\epsilon_0$ is the low-frequency dielectric constant, g_{eff} is the effective quadratic electro-optic coefficient, and V is the external voltage applied to the crystal between electrodes separated by distance l; $-q$ is the electron charge, k_B is Boltzmann's constant, and T is the temperature. Finally, we define the optical and the background intensities as $I = |A|^2$ and I_b, respectively, and the dark irradiance, I_{dark}, as β/s. Note that I_{dark} is very small compared with I and I_b, which are typically used in most photorefractive materials.[8,10]

We seek solutions of the form

$$A(x,z) = u(x)\exp(i\Gamma z)(I_{\mathrm{dark}} + I_b)^{1/2}, \tag{6}$$

where Γ is the soliton propagation constant. Here we discuss bright and dark solitons only; thus we limit our analysis to real $u(x)$. Since $I = |A|^2$ depends on x alone, we look for solutions in which $\hat{n}$, $N_d{}^i$, $\hat{\mathbf{J}}$, and $\hat{\mathbf{E}}$ depend solely on x and the only component of $\hat{\mathbf{E}}$ and $\hat{\mathbf{J}}$ is in the x direction.

We transform the equations to dimensionless form by the substitutions $n = \hat{n}/N_d$, $r = N_d/N_A$, $N = N_d{}^i/N_d$, $E = |\hat{\mathbf{E}}|/(V/l)$, $J = |\hat{\mathbf{J}}|/(q\mu N_d V/l)$, and $\xi = x/d$; $d = (\pm 2kb)^{-1/2}$ is the characteristic length scale and $b = (k/n_b)[1/2 n_b{}^3 g_{\mathrm{eff}}\epsilon_0{}^2(\epsilon_r - 1)^2(V/l)^2] = (k/n_b)\Delta n_0$ is the parameter that characterizes the strength and the sign of the optical nonlinearity. Notice that $d = \lambda/[2\pi(\pm 2\Delta n_0 n_b)^{1/2}]$, where Δn_0 is the change in the refractive index driven by the dc Kerr effect in the absence of light at a uniform external field V/l. We solve for a scalar case that occurs, for example, in KLTN when x is parallel to the direction of the applied field.[15,16] The sign of g_{eff} determines the sign (positive or negative) of Δn_0. We therefore introduce the dual-sign ($\pm$) notation in the definition of d, where the upper (lower) sign applies to the positive

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(negative) value of b (and, consequently, of Δn_0). The dimensionless equations are

$$n - a(1 + u^2)(1 - N)/N = 0\,, \tag{7}$$

$$J = nE + \epsilon_1 n' = \text{const.}\,, \tag{8}$$

$$(N - 1/r - n) - \epsilon_2 E' = 0\,, \tag{9}$$

$$l/d + \int_{-l/2d}^{l/2d} \mathrm{d}\xi E = 0\,, \tag{10}$$

$$u'' = \pm(\delta - E^2)u\,, \tag{11}$$

where $\delta = \Gamma/b$ and, unlike in the case of Ref. 8, Eq. (11) now depends on E^2. The prime and double prime stand for the derivatives with respect to ξ, $a = s(I_{\mathrm{dark}} + I_b)/(\gamma N_d)$, $\epsilon_1 = k_B Tl/(qdV)$, and $\epsilon_2 = V\epsilon_s/(lqdN_d)$. For typical parameters of KLTN ($\lambda \approx 0.5$ μm, $n_b \approx 2.2$, $\epsilon_s \approx 4000$ ϵ_0, $V/l = 2$ kV/cm, $N_d \approx 10^{18}$ cm^{-3}, $r \approx 20$), we obtain $\Delta n_0 \approx 5 \times 10^{-4}$ (which can trap a 10-μm soliton[8,10]), $d \approx 1.7$ μm, $\epsilon_1 \approx 0.074$, and $\epsilon_2 \approx 0.013$. In addition, with optical intensities of ≈ 1 W/cm^2, n is of the order of 10^{-9}. Note that ϵ_s depends on the temperature T via the Curie–Weiss law and can be safely set between 3000 and 10,000, as long as the operation point is far enough from the phase transition.

In view of these estimated values, n and $\epsilon_2 E'$ can be neglected in Eq. (9); thus $N = 1/r$, which is a constant (as for screening solitons) as long as E' is much smaller than $(r\epsilon_2)^{-1} \approx 4$. With this constant in Eq. (7) leads to $n \cong ar(1 + u^2)$, since $N \ll 1$. Similarly, in Eq. (8) one can neglect $\epsilon_1 n'$ with respect to nE, as long as $|\epsilon_1 n'| \ll 1$. One can easily justify both of these approximations *a posteriori* after solving for u and E (in fact, this has been shown for screening solitons and strontium barium niobate parameters in Ref. 7). Substituting n in Eq. (8) leads to $E = J/[ar(1 + u^2)]$, which, after substitution of E into Eq. (10) gives

$$E = -\eta/(1 + u^2)\,, \tag{12}$$

with $\eta = [(\mathrm{d}/l) \int_{-l/2d}^{l/2d} \mathrm{d}\xi/(1 + u^2)]^{-1}$. It remains now to evaluate constant η (which is proportional to the constant current J). This constant can be evaluated with high accuracy by use of the procedure described in the appendix of Ref. 8. Here we choose to evaluate this constant by use of a simpler method: assuming a square-wave structure for the optical beam. For bright solitons we assume a square wave of intensity u_0^2 and width Δx (in dimensional units), and for dark solitons we assume a square notch of width Δx in a uniform beam of intensity u_∞^2. Performing the appropriate integration yields $\eta \cong 1$ for bright solitons as long as $\Delta x/l \ll 1$ and $\eta \cong 1 + u_\infty^2$ as long as $1 + u_\infty^2 \ll l/x$. Typically, $l \approx 5$ mm, and the soliton width, Δx, is roughly 10 μm. This means that the approximate value of $\eta = 1$ for bright solitons is always valid, whereas for dark solitons $\eta = 1 + u_\infty^2$ holds for $1 + u_\infty^2 \ll 2500$ only. Note that our results imply Δn that is proportional to the screening field squared [and thus to $(V/l)^2$], as opposed to Δn used for two-wave-mixing that is proportional to the product of the unscreened field V/l and the diffusion field.[15,16] Here the diffusion field plays only a secondary role (being the first-order correction) and, as with screening solitons, we expect it to give rise to self-bending.[7]

We now substitute E into the normalized wave equation and obtain

$$u'' = \pm[\delta - \eta^2/(1 + u^2)^2]u\,, \tag{13}$$

which can be integrated by quadrature and yields

$$p^2 - p_0{}^2 = \pm\{\delta(u^2 - u_0{}^2) + \eta^2[(1 + u^2)^{-1} - (1 + u_0{}^2)^{-1}]\}\,, \tag{14}$$

with $p(\xi) = u'$, $p_0 = p(0)$ and $u_0 = u(0)$.

Fundamental bright solitons are found under the conditions[6,8] (i) $u(\infty) = u'(\infty) = u''(\infty) = 0$, (ii) $p_0 = 0$, and (iii) $u''(0)/u_0 < 0$. Substituting $\xi \to \infty$ and using conditions (i) and (ii) in Eq. (14) yields $\delta = 1/(1 + u_0{}^2)$, which also implies that $0 < \delta < 1$ for all nonzero u_0. Using this result in Eq. (13) and applying condition (iii) implies that only the lower sign can give bright solitons. This means that, similar to screening solitons,[6,8] bright solitons in centrosymmetric photorefractive media also require $\Delta n_0 < 0$ (everywhere) and can be viewed intuitively by Fig. 1 of Ref. 8. To summarize, bright solitons are solutions of

$$u'' = -\left[\frac{1}{1 + u_0{}^2} - \frac{1}{(1 + u^2)^2}\right]u\,, \tag{15}$$

with $u(0) = u_0$ and $u'(0) = 0$. We integrate Eq. (15) numerically and obtain the soliton waveforms that are qualitatively similar to those of screening solitons.[6–8] Note that, for $u_0 \ll 1$, both solitons are in the Kerr limit and attain a hyperbolic secant form. We obtain the soliton existence curve that relates the soliton intensity FWHM width $\Delta\xi$ to u_0 and show it as the solid curve in Fig. 1, in which we also show the existence curve of conventional screening solitons (dashed curve). The minimum of each curve is different and is obtained for $u_0 \cong 1.06$ (which gives $\Delta\xi \cong 3.34$) for the present case. The stronger saturation in the present case is apparent from Fig. 1: it is manifested in the faster increase in $\Delta\xi$ for u_0 values that are larger than the minimum.

Fundamental dark solitons are found under the conditions[6,8] (i) $u'(\infty) = u''(\infty) = 0$, (ii) $u_0 = 0$, and (iii) p_0

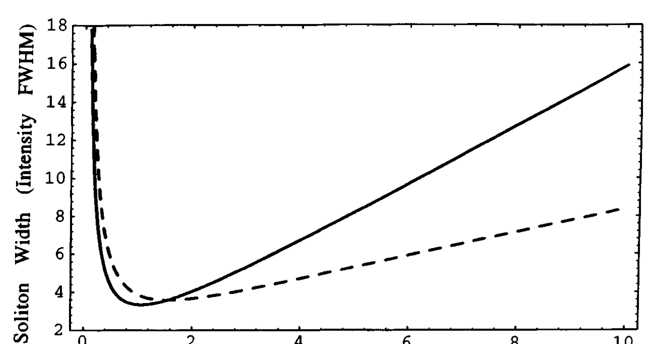

Fig. 1. Existence curves of bright solitons in photorefractive centrosymmetric media (solid curve) and of screening solitons in photorefractive noncentrosymmetric crystals (dashed curve).

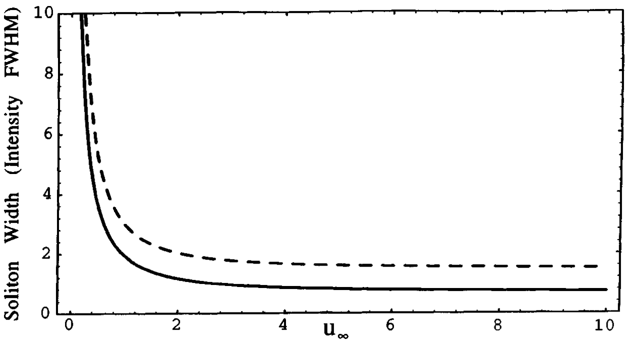

Fig. 2. Existence curves of dark solitons in photorefractive centrosymmetric media (solid curve) and of screening solitons in photorefractive noncentrosymmetric crystals (dashed curve).

must be real. Substituting $\xi \to \infty$ and condition (i) into Eq. (13) leads to $\delta = 1$ for all dark solitons. Thus, all the dark solitons propagate with a phase velocity identical to the light speed in the biased bulk outside the dark notch. Substituting now $\xi \to \infty$, $\delta = 1$, and conditions (i) and (ii) into Eq. (14) leads to $p_0{}^2 = \pm u_\infty{}^4$. Condition (iii) implies that only the upper sign gives dark solitons. Similar to screening solitons,[6,8] dark solitons in centrosymmetric photorefractive media require that $\Delta n_0 > 0$ and can be explained intuitively by Fig. 4 in Ref. 8. To summarize, dark solitons are solutions of

$$u'' = \{1 - [(1 + u_\infty{}^2)/(1 + u^2)]^2\}u\,, \tag{16}$$

with $u(0) = 0$ and $u'(0) = u_\infty{}^2$. We integrate Eq. (16) numerically and obtain the waveforms and the existence curve for dark solitons. In Fig. 2 we show the existence curve of dark solitons in centrosymmetric media (solid curve) and, for comparison, the existence curve of dark screening solitons (dashed curve). The curves have a similar trend, but the faster saturation of the present case reduces the asymptotic minimum at large u_∞ values to $\Delta\xi \cong 0.7$, which implies that the Δn_0 required for dark solitons in centrosymmetric media is reduced by roughly a factor of 4 compared with Δn_0 for dark screening solitons.

Several issues now merit discussion. First, solitons that result from $\Delta n \propto 1/(1 + |A|^2)^2$ have been studied theoretically,[17,18] and several intriguing predictions have been made, including stability with respect to small perturbations[17]; bistability,[17,19] which in the present case exists for bright solitons only (as is apparent from Fig. 1, solitons of the same width $\Delta\xi$ are found for two different values of u_0); and drift instability of dark solitons.[18] To our knowledge, no other physical system that exhibits this form of nonlinearity has been found. We expect that solitons in photorefractive centrosymmetric media will be good candidates for study of soliton properties in highly saturated nonlinearities, including collisions and other interactions. Second, as shown in Figs. 1 and 2, these new solitons require a smaller nonlinearity than screening solitons for support of a soliton of the same width and wavelength. Furthermore, the maximum attainable Δn_0 in these materials is very large,[15,16] and the space-charge field can be impressed (fixed) into the crystalline lattice. This means that one should be able to utilize the solitons to induce waveguides in these crystals and transform them into permanent waveguides with a highly controllable shape (tapered waveguides, Y junctions, etc.). The high saturation should also support stable two-dimensional self-trapping, as observed with screening solitons[10] and give rise to two-dimensional soliton-induced waveguides.

In conclusion, we have shown that centrosymmetric photorefractive media should support spatial solitons.

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