Soliton transverse instabilities in anisotropic nonlocal self-focusing media

Kristian Motzek and Friedemann Kaiser

Institute of Applied Physics, Darmstadt University of Technology, Darmstadt D-64289, Germany

Wen-Hen Chu and Ming-Feng Shih

Department of Physics, National Taiwan University, Taipei 106, Taiwan

Yuri S. Kivshar

Nonlinear Physics Group, Research School of Physical Sciences and Engineering, Australian National University, Canberra ACT 0200, Australia

Received August 13, 2003

We study, both numerically and experimentally, the transverse modulational instability of spatial stripe solitons in anisotropic nonlocal photorefractive media. We demonstrate that the instability scenarios depend strongly on the stripe orientation, but the anisotropy-induced features are largely suppressed for spatial solitons created by self-trapping of partially incoherent light. © 2004 Optical Society of America

OCIS codes: 190.4420, 190.5330.

Nonlinearity-driven instabilities have been studied in many different branches of physics, since they provided a simple means to observe strongly nonlinear effects in nature. Transverse (or symmetry-breaking) instabilities of solitary waves were predicted theoretically almost 30 years ago,[1] but only recently both transverse and spatiotemporal instabilities were observed for different types of bright and dark spatial optical solitons.[2,3]

Many of the experimental studies of spatial optical solitons in a nonlinear bulk medium are being carried out for photorefractive crystals that are known to exhibit an anisotropic nonlocal response characterized by an asymmetric change of the refractive index. Given the strong anisotropy of the photorefractive nonlinearities, it is crucially important whether the theoretical results obtained mostly for isotropic nonlinear media[2] can be applied, at least qualitatively, to the case of anisotropic nonlocal media. In particular, the transverse instabilities of solitary waves that develop under the action of higher-order perturbations or temporal effects are known to initiate a breakup of a stripe soliton, and several different scenarios of the breakup dynamics are known.[2] The most interesting scenario is the generation of new stable localized structures. In the scalar case, this corresponds to a breakup of a soliton stripe into (2 + 1)-dimensional bright solitons in a self-focusing medium.

The purpose of this Letter is twofold. First, we employ a simple model of an anisotropic nonlocal medium that takes into account important properties of photorefractive nonlinearities[4] and demonstrate numerically that the classical scenario of the soliton transverse instability, namely, the stripe breakup and formation of two-dimensional spatial solitons,[2] depends dramatically on the stripe orientation; these results are fully confirmed by our experimental studies of the breakup of vertical, horizontal, and tilted soliton stripes. Second, we generalize the familiar coherence-function approach, which is employed for describing the propagation of partially incoherent light in nonlinear media, and study the transverse instability of partially incoherent soliton stripes in anisotropic nonlocal media. We demonstrate theoretically and confirm experimentally that strong anisotropy-driven features of photorefractive nonlinearity are largely suppressed by spatial partial incoherence of light.

We consider the propagation of a single optical beam with slowly varying amplitude E in a biased photorefractive crystal, described by the paraxial equation

$$i\,\frac{\partial E}{\partial z} + \frac{1}{2}\,\nabla_{\perp}{}^{2}E = \frac{\partial \varphi}{\partial x}\,E\,, \tag{1}$$

where $\nabla_{\perp}$ stands for the transverse gradient in the plane perpendicular to the propagation direction z and $\partial\varphi/\partial x$ yields the electric field inside the crystal. We assume that an external electric field is applied to the crystal, and it is parallel to the direction of x, so that electric potential φ is defined by the potential equation[4]

$$\nabla_{\perp}{}^{2}\varphi + \nabla_{\perp}\varphi\nabla_{\perp}\ln(1+I) = \frac{\partial}{\partial x}\ln(1+I)\,, \tag{2}$$

where $I = |E|^2$ is the light intensity inside the crystal.

First, we study numerically the nonlinear evolution of a narrow stripe oriented perpendicular to the external electric field. Figures 1a–1c show images of the nonlinear evolution at two different propagation distances (b and c). In numerical simulations, the initial diameter of the input Gaussian beam was chosen to be close to that of a solitary solution, and white noise (~2%) is added to the screening field. Increasing nonlinearity (i.e., the applied field) leads

0146-9592/04/030280-03$15.00/0

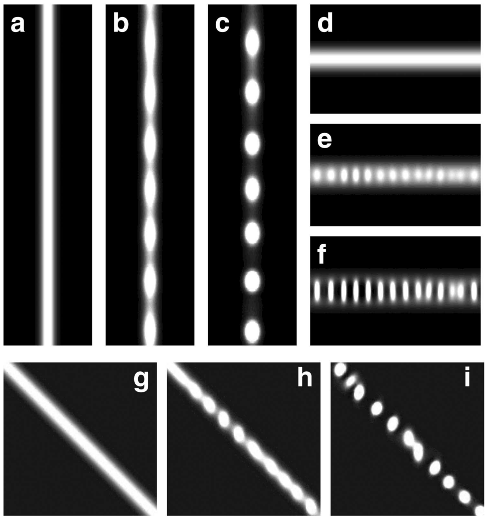

Fig. 1. Numerical results for the coherent-stripe propagation: a–c, the soliton stripe is perpendicular to the external electric field; d–f, the soliton stripe is parallel to the field; g–i, an intermediate orientation of a tilted stripe.

to a breakup of the vertical stripe as a result of the transverse modulation instability.[5,6] At the initial stage of that breakup all spatial harmonics of the noise are small and each is amplified exponentially with its own growth rate. The fastest-growing modes become noticeable first and at later stages determine the characteristic spatial scale of the breakup. At larger propagation distances (larger nonlinearities) the beam transforms into an array of (2 + 1)-dimensional bright solitons (see Fig. 1c).

Figures 1d–1f show the evolution of a stripe parallel to the external field. Because solitary solutions of this form do not exist, the stripe initially diffracts. Subsequently, a growing spatial modulation, Fig. 1e, eventually breaks up the stripe into vertical elongated filaments, Fig. 1f, but not solitons. Figures 1g–1i show the same effect for the intermediate orientation of the stripe.

We also studied numerically the propagation of partially incoherent light stripes in an anisotropic photorefractive medium described by model equations (1) and (2). We have extended the coherent density approach[7] used earlier for the study of isotropic media (see, e.g., Ref. 8) and described the anisotropic nonlocal response according to Eq. (2). The coherent density approach is based on the fact that partially incoherent light can be described by a superposition of mutually incoherent light beams that are tilted with respect to the z axis at different angles; i.e., the partially incoherent light stripe consists of many coherent but mutually incoherent light stripes E_j: $I = \sum_j |E_j|^2$. By setting $|E_j|^2 = G(\vartheta)I$, where $G(\theta) = (\pi^{1/2}\theta_0)^{-1}\exp(-\theta^2/{\theta_0}^2)$ is the angular power spectrum, we obtain a partially incoherent light stripe whose coherence is determined by the parameter θ_0; i.e., less coherence means bigger θ_0. Here, $j\vartheta$ is the angle at which the jth beam is tilted with respect to the z axis. If the light is incoherent along the y axis (vertical stripes), they are tilted in the y direction, whereas they are tilted in the x direction if the light is to be incoherent along that axis (horizontal stripes).

Figures 2a and 2b show our numerical results for the propagation of a vertical stripe close to the stability threshold with the degree of incoherence determined by $\theta_0 = 0.43°$. The external electric field is perpendicular to the stripe. The most obvious difference to the scenario of the coherent-stripe decay is that the filaments are much longer. Furthermore, the filaments change their profile only very slowly as they propagate and thus can be considered as incoherent solitons. Larger values of θ_0 (i.e., $\theta_0 > 0.43°$) correspond to a complete suppression of the soliton transverse instability. This corresponds to the existence of a threshold for incoherent modulational instability. This corresponds to the existence of a threshold for incoherent modulational instability[9] and soliton transverse instability[10] in an isotropic medium.

Figures 2c and 2d show the propagation of an incoherent light stripe parallel to the external electric field with $\theta_0 = 0.56°$. Increasing the incoherence further leads to the case in which the beam diffracts before the transverse instability can set in. Obviously the filaments have the same size as in the coherent case, but there is more intensity lost to radiation. This confirms previous results showing that it is very difficult to obtain solitary structures that are elongated along the axis of the external field.

We performed a number of experiments to study the development of the anisotropy-driven soliton transverse instability in photorefractive crystals, for both coherent (see Fig. 3) and partially incoherent (see Fig. 4) soliton stripes. The experiments were conducted with a photorefractive SBN:61 crystal in a setup similar to that employed for observation of self-trapping of partially incoherent light.[11] We also illuminated the entire crystal with a background light with an intensity much stronger than that of the soliton stripe to make the nonlinearity close to a Kerr-type self-focusing nonlinearity.

The beam was made spatially incoherent by passing it through a rotating diffuser. The rotating diffuser

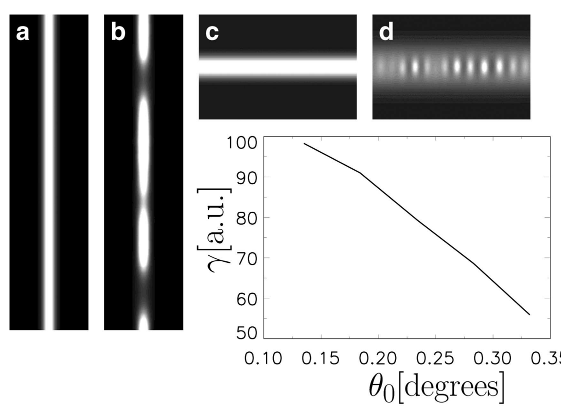

Fig. 2. Numerical results for the partially incoherent stripes. In a and b the stripe is perpendicular to the external field, and in c and d the stripe is parallel to the field. Shown are the (a and c) input and (b and d) output beams. The plot shows the growth rate of the transverse instability versus coherence parameter θ_0.

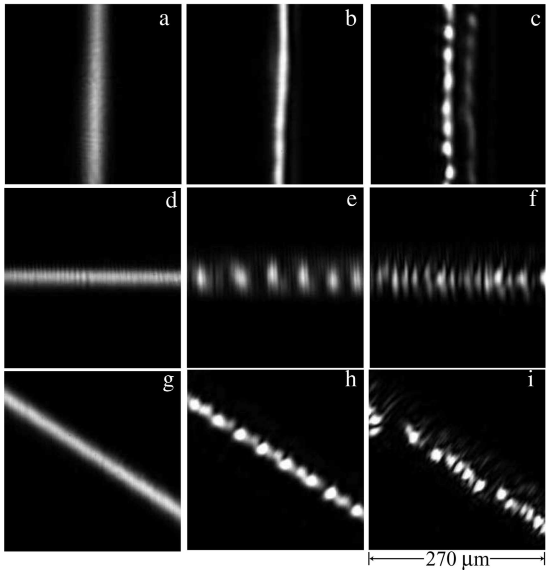

Fig. 3. Experimental observation of the transverse instability of a coherent soliton stripe. a, d, g, inputs for three orientations of the stripes. All other images are taken at the crystal output (length 7 mm) for applied voltage of b, h, 1 kV; c, i, 2 kV; c, 0.5 kV; and f, 1 kV.

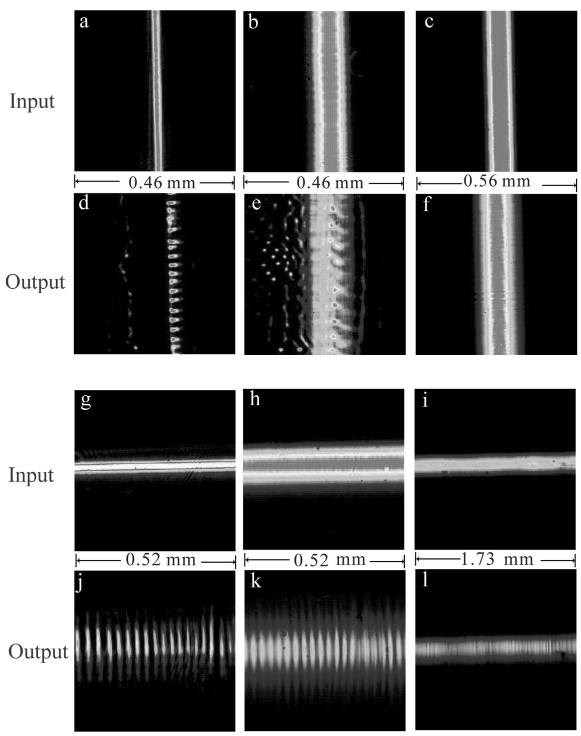

Fig. 4. Experimental results for the stripes oriented (a–f) perpendicular and (g–l) parallel to the electric field. a, g, input and d, j, output of the coherent light propagation (7 mm), presented for comparison; the other images correspond to partially incoherent light.

provided a new phase and amplitude distribution every 1 ms, which was much shorter than the response time (~1 s) of the medium. We followed Anastassiou *et al.*[8] and generated a beam that was narrow and fully coherent in the x direction, yet uniform and partially incoherent in the y direction.

Figures 3a–3c and 4a and 4d show the development of the transverse instability for the coherent stripe perpendicular to the electric field direction. As the degree of incoherence grows, the instability weakens (see Figs. 4b and 4e) and then disappears completely (see Figs. 4c and 4f) because, for one to observe the instability, the value of the nonlinearity has to exceed a threshold imposed by the degree of spatial coherence.[8] The nonlinearity is turned on by applying a voltage of 3 kV to the photorefractive crystal with $r_{33} = 250$ pm/V. Figures 3d–3f and 4g–4l show the development of instability for the stripe parallel to the field. An important observation is that the strong anisotropy-driven effects observed for coherent light are largely suppressed when the degree of spatial incoherence exceeds a threshold (see Figs. 3f and 4l).

In conclusion, we have studied theoretically and experimentally the transverse modulational instability of coherent and partially incoherent soliton stripes in photorefractive crystals. We have demonstrated a number of novel, anisotropy-driven features of the stripe instability and analyzed the effect of partial incoherence.

Y. Kivshar (e-mail, ysk124@rsphysse.anu.edu.au) thanks Glen McCarthy for discussions and the Department of Physics of Taiwan University for their hospitality. This work was partially supported by the Australian Academy of Science and the National Science Council, Taiwan.

References

1. V. E. Zakharov and A. M. Rubenchik, Zh. Eksp. Teor. Fiz. **65,** 997 (1973) [Sov. Phys. JETP **38,** 494 (1974)].
2. See Yu. S. Kivshar and D. E. Pelinovsky, Phys. Rep. **331,** 117 (2000).
3. Yu. S. Kivshar and G. P. Agrawal, *Optical Solitons: from Fibers to Photonic Crystals* (Academic, San Diego, Calif., 2003).
4. See, e.g., A. A. Zozulya and D. Z. Anderson, Phys. Rev. A **51,** 1520 (1995).
5. K. Kos, H. Meng, G. Salamo, M. Shih, M. Segev, and G. C. Valley, Phys. Rev. E **53,** R4330 (1996).
6. A. V. Mamaev, M. Saffman, D. Z. Anderson, and A. A. Zozulya, Phys. Rev. A **54,** 870 (1996).
7. D. N. Christodoulides, T. H. Coskun, M. Mitchell, and M. Segev, Phys. Rev. Lett. **78,** 646 (1997).
8. C. Anastassiou, M. Soljačić, M. Segev, E. D. Eugenieva, D. N. Christodoulides, D. Kip, Z. H. Musslimani, and J. P. Torres, Phys. Rev. Lett. **85,** 4888 (2000).
9. M. Soljačić, M. Segev, T. Coskun, D. N. Christodoulides, and A. Vishwanath, Phys. Rev. Lett. **84,** 467 (2000).
10. J. P. Torres, C. Anastassiou, M. Segev, M. Soljacic, and D. N. Christodoulides, Phys. Rev. E **65,** 015601 (2001).
11. M. Mitchell, Z. Chen, M. Shih, and M. Segev, Phys. Rev. Lett. **77,** 490 (1996).